

#### Objective

The goal is to draw the last curve (sprig) joining a set of dots.

Rules:

- 1. Take turns drawing curves connecting dots.
- 2. A curve must start and end at a dot. It could be the same dot.
- 3. Curves may not touch dots or each other, except at endpoints.
- 4. No dot may have more than two curve ends touching it.

#### Materials

- 1. Dry erase boards and markers
  - a. Alternatively, you could have students use paper and pencil.

Optional: Sprigs <u>instructions</u> p. 6 Sprigs <u>tasks</u> p. 7 Sprigs <u>extensions</u> p. 8

#### **Introduction (5 minutes)**

#### Excite

Draw 5 dots on the board and challenge the class to beat you at drawing the last curve.

#### Explain

- 1. Explain that the curves must start and end a dot and that curves can't touch. Draw the first curve and ask students for suggestions for their turn.
- 2. On your second turn, draw a curve that starts and ends at the same dot. Ask if they think that's allowed (yes). Ask why (a curve must start and end at a dot it doesn't say it has to be different dots).
- 3. On your third turn, draw a curve that crosses another curve. Ask if they think that's allowed (*no*). Ask why (*curves cannot touch*).
- 4. Continue taking turns until every dot is connected and declare a winner.
- 5. Have students count the number of curves out loud.



6. Have the students count the number of curve ends at each dot.

#### Engage

- 1. Explain that you're going to play again but with one more rule. This time each dot can have, at most, two curve ends touching it.
- 2. Take turns drawing curves according to the new rule. And the winner is whoever can draw the last curve.

#### **Common misconceptions**

Students might think that:

- a. A curve that starts and stops at the same dot counts as one curve end.
- b. Starting with a lot of dots will be more fun to explore.

Explore (30 minutes)

In pairs, have the students explore drawing dots and curves.

Encourage them to start smaller (3 dots, 4 dots...) and see if they can decide if winning depends on whether you go first or second.

After about 5 minutes, gather together again whole group.

Draw a table on the board labeled:

<u>2-sprigs - # of curves (moves) - which player wins</u>

2 dots 3 dots 4 dots...

Have the students help you populate the table. You may have to recreate example games for them to count with you.

Tell them you want to add a new rule. This time the most curve ends you can have at a dot is 3. Play a 3-dot game using their suggestions and together count the curve ends.

Create a new 3-sprigs table and add the info from the game they just played with you.

Have them work in partners to explore the 3-sprigs game. Encourage them to keep track of their discoveries by creating a table or some other method.

Circulate and ask questions to encourage deeper thinking:

- a. Are there different kinds of first moves you can make? Are any of them better than the others?
- b. At what point during the game were you sure who would win? How did you know?
- c. Is it better to play first or second? Does this depend on the number of dots?
- d. How many moves are there with 2 dots? 3 dots?...
- e. Do any of the regions have a shape that tells you something interesting?

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f. "Tell me more." is a great basic prompt for getting a child to explain their thinking.

#### **Discussion (5-10 minutes)**

As a whole group, have students share something about their experience with Sprigs. Try to have at least 3 students share out. Variations of the questions asked earlier are great for generating discussion, such as:

- a. What kinds of first moves did you make?
- b. Is it better to play first or second?

#### Assessment

Evidence of student learning during problem-solving activities can be obtained from three sources: observations, conversations, and products.

**Observation** involves actually observing students while they perform tasks and demonstrate skills and may take the form of a checklist or quick note.

**Conversation** involves engaging students in discussion that encourages them to articulate what they are thinking and then capturing that with a quick note.

**Products** are student-created records that capture not only their answer, but some of the process that led them to the answer.

#### **Standards**

- 1. Make sense of problems and persevere in solving them. CCSS.MP1
- 2. Construct viable arguments and critique the reasoning of others. CCSS.MP3
- 3. Look for and express regularity in repeated reasoning. CCSS.MP8

#### Answers

These answers are to give you confidence while doing the activity. However, the real answers are in the way that students solve the puzzles and explain their thinking.

#### **General Answers:**

- 1. **2-Sprigs**, ends in *n* moves, where *n* is the number of vertices (dots).
- 3-Sprigs, the maximum number of moves is 3n/2, where n is the number of dots. (This is rounded down when n is odd). However, 3-Sprigs does not necessarily need to end in [3n/2] moves, (see 2c).

3 sprigs	Min # of moves	Max # of moves	Who wins
2 dots	3	3	1st player
3 dots	4	4	2nd player
4 dots	5	6	2nd player
5 dots	6	7	1st player
6 dots	8	9	2nd player

a. 3-sprigs & 2 dots - ends after 3 moves. There are two possible endings:



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b. 3-sprigs & 3 dots - ends after 4 moves. There are five possible endings.



OR

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c. 3-sprigs & 4 dots - the second player can always win.We see the first game that doesn't always end after a fixed number of moves.



d. 3-sprigs & 5 dots - the first player can always win.





# **Sprigs Instructions**

Draw a few dots and take turns drawing curves connecting the dots.

Rules:

• A curve must start and end at a dot. It could be the same dot.





• Curves may not cross over each other.



• No dot may have more than two curve ends touching it.



The winner is the player who draws the last curve.

## **Sprigs Tasks**

1. Start with three dots and play a few games.

What are some first moves that can be made?

Does it matter who goes first?

Does it matter how you place the dots on the paper?



2. What if you have 4 dots?



3. 5 or more dots?



## **Sprigs Challenge**

Draw four dots and play again, but this time with a slight rule change:

New Rule:

Each dot may be connected to no more than *three* curve ends.



1. Start with four dots and play a few games.

Does it matter who goes first?

What's the fewest moves it takes to win?

What's the least moves it takes to win?

2. What if you have 5 dots? 6 or more dots?



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