

Objective

How to safely carry wolves, goats, and cabbages across a river by boat.

• The number of wolves, goats, and cabbages being transported will vary from puzzle to puzzle, as will the number of items the boat can carry.

Rules:

- 1. Wolves and goats can't be left together a wolf will eat a goat.
- 2. Cabbages and goats can't be left together a goat will eat a cabbage.

Materials

- 1. Counters in three different colors that can be written on.
 - a. 10 of each color to represent wolves, goats, and cabbages.
 - b. Alternatively, have students write W, G, and C on colored slips of paper.
- 2. River Crossings task sheets pp. 6-8
 - Note: For many students, it can be less intimidating to have each Puzzle/Question printed on a separate piece of paper. Keep them centrally located in the room and students can return completed puzzles and pick up the next one.

Optional: Small container or index card to represent the boat River Crossings <u>extensions sheet</u> p. 9 River Crossings <u>river mat p. 10</u>

To explore the activity yourself, you can try our digital version here: <u>jrmf.org/puzzle/river-crossings</u>

Introduction (5 minutes)

Excite:

Tell students that River Crossings is one of the oldest, well-known puzzles in human history. It's over 1000 years old and has been found throughout the world: in Africa, Europe, Asia, and even in the Simpsons!

Explain:

- 1. You will be solving Puzzles 1 and 2 as a whole group, but don't show them how to solve it! Instead, ask students to help guide you through the puzzle.
- 2. Using the colored counters (G, W, C) introduce Puzzle 1 by telling a story about wanting to help a wolf, goat, and a cabbage cross a river or, even better, get three volunteers to



act out the scenario with you.

- 3. Explain that you can only take one item at a time in the boat with you and that all the items are safe when you are nearby.
- 4. Demonstrate the rules for Puzzle 1 by pretending to take the cabbage across first the wolf then eats the goat!
- 5. Try again, this time taking the wolf the goat then eats the cabbage!
- 6. Ask the students what else they could try (*goat*) and demonstrate taking the goat safely across.

Engage:

- 1. Ask students to help guide you to finish Puzzle 1. If students aren't sure what to do, encourage them to experiment and try something new. A key understanding that needs to emerge here is that items can go back and forth across the river.
- 2. Introduce Puzzle 2, which is an impossible puzzle. Let students struggle to help you with this until someone questions whether it's possible. This is a great AHA moment not all of these puzzles will have a solution and that's okay.
- 3. Point out that sometimes you can carry more than one item in the boat!

Common misconceptions

Students might think that:

- a. They can't bring the same item across the river more than once.
- b. All items are safe once they are on the opposite shore.

Explore (30 minutes)

Have the students work through the rest of the puzzles in pairs. Students will physically move counters back and forth across their desk (representing the river).

Circulate and ask questions to encourage deeper thinking:

- a. What if you had one more [insert wolf, goat, or cabbage here]? How would that have changed the puzzle?
- b. Which puzzle did you find the hardest so far? The easiest? Why?
- c. Have you found a strategy that works for all or many of the puzzles?
- d. When a student is stuck, ask:
 - i. Are there times where you had to choose between options?
 - ii. What do you know so far?
 - iii. What didn't work?
 - iv. What are you thinking about trying?
- e. To support pattern recognition, ask:
 - i. How can I tell what moves you made?
 - ii. What do you think will happen?
 - iii. Is there a way to organize what we know to understand it better?

Encourage students to develop their own method for keeping track of their work. Some students may use arrows, some may use letters, and some may use other methods; all of

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these are acceptable as long as they help students remember their work!

Extend

- 1. Encourage students to create their own solvable River Crossings puzzles.
- 2. Give students the River Crossings extension (Zombies and Humans).

Discussion (5-10 minutes)

As a whole group, have students share something about their experience with River Crossings. Try to have at least 3 students share out. Variations of the questions asked earlier are great for generating discussion, such as:

- a. Which was your favorite puzzle? Why?
- b. Which puzzle did you find the hardest? Easiest? Most interesting? Most surprising? Why?
- c. What strategies did you come up with? Did they always work?

Assessment

Evidence of student learning during problem-solving activities can be obtained from three sources: observations, conversations, and products.

Observation involves actually observing students while they perform tasks and demonstrate skills and may take the form of a checklist or quick note.

Conversation involves engaging students in discussion that encourages them to articulate what they are thinking and then capturing that with a quick note.

Products are student-created records that capture not only their answer, but some of the process that led them to the answer.

Standards

- 1. Make sense of problems and persevere in solving them. CCSS.MP1
- 2. Construct viable arguments and critique the reasoning of others. CCSS.MP3
- 3. Model with mathematics. CCSS.MP4
- 4. Look for and make use of structure. CCSS.MP7

Answers

These answers are to give you confidence while doing the activity. However, the real answers are in the way that students solve the puzzles and explain their thinking.

General Answers:

For wolves, goats, and cabbages, if you have a boat that can carry 2 items:

- You can have 1 goat and any number of other non-goats.
- You can have 2 goats and take a maximum of 4 non-goats.
- You can have 4 goats and take a maximum of 2 non-goats.

For humans and zombies:

- If the boat can hold 2 individuals, then 3 of each can cross.
- If the boat can hold 3 individuals, then 4 of each can cross.
- If the boat can hold 4 individuals, any same number of each can cross.

Possible Student Answers to Questions 1 to 7:

- 1. Try puzzles 1 and 2. One of these two puzzles is impossible. What makes it impossible?
 - a. Puzzle 2 is impossible because, if you can only carry one item at a time, you can't have more than one goat. It would work if there was only one wolf or only one cabbage.
- 2. Try puzzle 3. Was puzzle 3 possible or impossible? Why do you think puzzle 3 is easier to figure out than puzzle 2?
 - a. Puzzle 3 is possible. It's easier because you can take all of one item (goats or non-goats) at the same time.
- 3. Try puzzles 4, 5, and 6. One of these puzzles is impossible. Which one is it? Of the two puzzles you could solve, which one was the hardest to solve? What made it so hard?
 - a. Puzzle 6 is impossible. Puzzle 4 is the hardest because once you know the strategy (carry the non-goats back and forth each time), you can use it for Puzzle 5.
- 4. Try puzzles 7 and 8. What strategy did you use to solve puzzles 7 and 8? How would you explain this strategy to a friend?
 - a. You can move one goat at a time by keeping the wolf and cabbage on the boat at all times.
- 5. Make a prediction. Can you predict if Puzzle 9 is possible or impossible before trying to solve it? If you think it's possible, how would you solve it? If you think it's impossible, how would you explain why to a friend?
 - a. It is possible because you can keep the wolf and cabbage on the boat the entire time and move one goat at a time.

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- 6. Try puzzles 10, 11 and 12. What strategy did you use for these puzzles? If you needed to change your strategy from previous puzzles, how did you do so?
 - a. Puzzle 11 is impossible. For puzzles 10 and 12, the strategy is to move the two goats back and forth each time.
- 7. Make a prediction. One of these puzzles is impossible. Without trying to solve them, can you predict which of these three puzzles is impossible? Why do you think it's impossible?
 - a. Puzzle 14 is impossible. If you have 3 goats, you can only bring a maximum of 6 non-goats.

Possible Student Answers to Extension Questions:

- 1. What if humans and zombies were crossing the river?
 - a. Can 3 humans and 3 zombies cross the river if the boat can hold 2 items?
 - This can be solved in 11 crossings.
 - b. Can 4 humans and 4 zombies cross the river if the boat can hold 2 items?
 Unsolvable. Need a minimum of 3 seats in the boat.
 - c. Can 5 humans and 4 zombies cross the river if the boat can hold 2 items?
 Unsolvable. Need a minimum of 3 seats in the boat.
 - d. Can 5 humans and 5 zombies cross the river if the boat can hold 2 items?
 Unsolvable. Need a minimum of 4 seats in the boat.
 - e. Can 6 humans and 5 zombies cross the river if the boat can hold 2 items?
 - Unsolvable. Need a minimum of 4 seats in the boat.
 - f. Can 6 humans and 6 zombies cross the river if the boat can hold 2 items?
 - Unsolvable. Need a minimum of 4 seats in the boat.

River Crossings Activity Instructions and Puzzles

How would you help a wolf, a goat, and a cabbage cross the river?

Rules:

- The number on the side of the boat tells how many items it can carry.
- You can't leave the wolf and goat together, since wolves eat goats.
- You can't leave the goat and cabbage together, since goats eat cabbages.

Question 1

One of these two puzzles is impossible. What makes it impossible?



Question 2

Try Puzzle 3. Now you can carry an extra item. Is it possible or impossible?

Why do you think puzzle 3 is easier to figure out than puzzle 2?



Question 3

Try Puzzles 4, 5, and 6. One of these puzzles is impossible. Which one is it?

Of the two puzzles you could solve, which one was the hardest to solve? What made it so hard?



Question 4

Try Puzzles 7 and 8. What strategy did you use to solve these puzzles? How would you explain this strategy to a friend?





Can you predict if Puzzle 9 is possible or impossible before trying to solve it?

If you think it's possible, how would you solve it? If you think it's impossible, how would you explain why to a friend?



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Question 6

Try Puzzles 10, 11, and 12. What strategy did you use for these puzzles? If you needed to change your strategy from previous puzzles, how did you do so?



Question 7

One of these puzzles is impossible. Without trying to solve them, can you predict which of these three puzzles is impossible?



Why do you think it's impossible?



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River Crossings Challenge

What if humans and zombies were crossing the river?

- At least one human or zombie needs to be in the boat to row from one side to the other.
- If there are ever more zombies than humans on one side of the river (on the shore and in the boat), then the zombies will eat the humans.



1. Which of these zombie and human puzzles can be solved with a boat that can hold 2 items? For those that can't be solved, what's the smallest boat you need?







