

Map Coloring Teaching Guide

Objective

Color the map, trying to use as few colors as possible.

Rule:

• Neighboring states (or counties) may not be colored the same.

Introduction

Ask students to think about maps they've seen (or show some examples). What do they notice about how the mapmakers make regions (countries, states, provinces etc) stand out from each other?

Lead them to the idea of using color for distinction and mention that the fewest number of colors needed to color a map was a long-standing problem in mathematics until it was solved in 1976 using brand new technology - a computer!

Tell the students that they'll be doing some of their own map coloring to see just how many colors are needed!

Explain

- 1. Project the images on the Instructions sheet.
- 2. Point out examples where borders touch and where they don't.

Engage

Color the first map together using the students' suggestions, but ensuring that the map coloring rules are followed. Encourage them to explain their thinking out loud.

Common misconceptions

Students might think that:

• Regions meeting at a corner are neighboring, but states must share a length of border to be considered adjacent.

Exploration

In pairs, have the students color the maps using as few colors as possible.

Circulate and ask questions to encourage deeper thinking:

- a. Which maps did you find the hardest so far? The easiest? Why?
- b. Have you found a strategy that works for all or many of the maps?
- c. Are there maps that force you to use more colors? Why?



- d. Is introducing new colors only when you get cornered a good strategy or is it better to pick some number of colors in advance and stick to them?
- e. When a student is stuck, ask:
 - i. Is there a region you should start coloring first? Why?
 - ii. What do you know so far?
 - iii. What didn't work?
 - iv. What are you thinking about trying?
- f. "Tell me more." is a great basic prompt for getting a student to explain their thinking.

Discussion

As a group, have students share something about their experience with Map Coloring. Try to have at least 3 students share out. Variations of the questions asked earlier are great for generating discussion, such as:

- a. How did you decide what region to color first?
- b. What other strategy(s) did you use?
- c. What was the most colors you used? The fewest?

Materials

Map coloring <u>tasks</u> pp. 7-10 Crayons (or dry erase sleeves and markers) Map Coloring <u>Instructions</u> (to project) p. 6

To explore the activity yourself, you can try our digital version here: <u>jrmf.org/activities/map-coloring</u>

Assessment

Evidence of student learning during problem-solving activities can be obtained from three sources: observations, conversations, and products.

Observation involves actually observing students while they perform tasks and demonstrate skills and may take the form of a checklist or quick note.

Conversation involves engaging students in discussion that encourages them to articulate what they are thinking and then capturing that with a quick note.

Products are student-created records that capture not only their answer, but some of the process that led them to the answer.

Standards

- 1. Make sense of problems and persevere in solving them. CCSS.MP1
- Construct viable arguments and critique the reasoning of others. CCSS.MP3
- 3. Model with mathematics. CCSS.MP4
- 4. Look for and make use of structure. CCSS.MP7

General Answers:

No more than four colors are required to color the regions of any map on a plane so that no two adjacent regions have the same color. (Children don't need to know this, but this is called the four-color theorem - the first major theorem to be proved using a computer.)

Many simpler maps can be colored using two or three colors. The fourth color is needed for some maps, such as one in which one region is surrounded by an odd number of other regions, which touch each other in a cycle.

A couple substructures of maps that tell you something about potential colorings are cycles and wheels.

• A **cycle** of k regions is k regions where region 1 neighbors 2, 2 neighbors 3, 3 neighbors 4, ..., k-1 neighbors k, and k neighbors 1. If k is odd, then it will take at least three colors to color it. If k is even, two might suffice.



Example of 4-cycle (pink), 3-cycle (green), and 6-cycle (blue)

• A **wheel** of k+1 regions is a cycle of k regions plus an extra region that neighbors all k of them. If k is odd, then it will take at least four colors to color it. If k is even, three might suffice.



Examples of wheel with 6 spokes (yellow and red) and 8 spokes (blue and green)

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Puzzle Solutions:













Question 7





Map Coloring Instructions

Color each map, trying to use as few colors as possible.

Rule:

States (or counties) that share a border must be different colors.





Map Coloring Tasks

1. Color the maps below. What is the least number of colors you need?



2. Color the maps below. What is the least number of colors you need?



3. We added some states to the maps on the previous page. Color each of these new maps. Did you color anything differently this time?



4. Color the maps below. Is one of these maps easier to color than the other? (Counties that only share a corner can be colored the same color.)



Northwest Iowa Counties

Southeast Iowa Counties

5. Color the maps below. Is one of these maps easier to color than the other?



6. Color the maps below. Which map needs more colors? What's different about the map that needs more colors?



North USA

East USA

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7. Color the Contiguous United States using only four colors. Try to use the fourth color as few times as possible. Where do you need to use the fourth color?

