

Objective

Place as many numbered ladybugs on a leaf as possible.

Rule:

• Two ladybugs cannot add up to a third ladybug on a leaf.

Introduction

Have ready to go Ladybugs #1 to #6, the task sheets and the leaf mat.

Explain

Tell the students that each numbered tile represents a ladybug and the number of spots on her shell.

Engage

- 1. Model the rules by laying two tiles on the leaf and asking the students to find the sum of the two ladybugs.
- 2. Tell the students that more ladybugs want to join, but another ladybug can only land on the leaf if the number on her shell *doesn't equal* the sum of any two ladybugs already there.
- 3. Ask which ladybug can't land there. Encourage them to explain their thinking out loud.
- 4. Ask if they can think of a ladybug that could land there. Encourage them to explain their thinking out loud.
- 5. Ask which other ladybugs could be placed on the leaf.

Common misconceptions

Students might think that:

- a. On one leaf, the sum of two ladybugs can't equal the sum of *two* other ladybugs.
- b. The maximum number of ladybugs achieved by one set of choices cannot be superseded by making different choices.

Exploration

In pairs and starting with Ladybugs 1 to 6, have the students explore which ladybugs can be placed together. Next, have them 9 ladybugs on a single leaf. For a challenge, they can try placing 6 or more ladybugs on *two* leaves.

Circulate and ask questions to encourage deeper thinking:

a. What combinations does Ladybug 6 need to watch out for? Ladybug 7? 8? 9?



- b. What do you notice about placing odd ladybugs?
- c. What do we know about the sum of two odd ladybugs?
- d. When a child is stuck, ask:
 - i. Are there any ladybugs you can't use?
 - ii. What do you know so far?
 - iii. What didn't work?
 - iv. What are you thinking about trying?
- e. To support pattern recognition, ask:
 - i. Is it easier to place the largest ladybugs or the smallest ladybugs?
 - ii. Can we ask a simpler question?
 - iii. What do you think will happen?
 - iv. Is there a way to organize what we know to understand it better?
- f. "Tell me more." is a great basic prompt for getting a child to explain their thinking.

Extend

Have the student pairs try the challenges on page 8.

Discussion

As a group, have students share something about their experience with Ladybugs. Try to have at least 3 students share out. Variations of the questions asked earlier are great for generating discussion, such as:

- a. How do you start a puzzle? What do you do first?
- b. What strategies did you come up with? Did they always work?
- c. What did you do when you had to choose between different options? How did you decide?
- d. Did you keep track of places you didn't want to put a particular ladybug?
- e. What did you notice about placing odd ladybugs?

Materials

Counters numbered 1 to 9 (<u>purchase</u> or print on cardstock) Ladybugs <u>leaf mats</u> p. 9-10

Optional: Ladybugs <u>tasks sheet</u> p. 7 Ladybugs <u>instructions sheet</u> p. 6 Ladybugs <u>extensions sheet</u> p. 8

To explore the activity yourself, you can try our digital version here: <u>jrmf.org/activities/ladybugs</u>

Assessment

Evidence of student learning during problem-solving activities can be obtained from three sources: observations, conversations, and products.

Observation involves actually observing students while they perform tasks and demonstrate skills and may take the form of a checklist or quick note.

Conversation involves engaging students in discussion that encourages them to articulate what they are thinking and then capturing that with a quick note.

Products are student-created records that capture not only their answer, but some of the process that led them to the answer.

Standards

- Make sense of problems and persevere in solving them. CCSS.MP1
- 2. Construct viable arguments and critique the reasoning of others. CCSS.MP3
- 3. Model with mathematics. CCSS.MP4
- 4. Look for and make use of structure. CCSS.MP7

General Answers

For a single leaf, you can always place at least half of the ladybugs.

- Start by placing the first two consecutive numbers whose sum is greater than the largest number in the set.
 - For example, using Ladybugs 1 to 6: The largest number in the set is 6. The first two consecutive numbers whose sum is greater than 6 are 3 and 4 so they can be placed on the leaf.
- Then put on the leaf all the remaining ladybugs that are larger than the two already placed.
 - For example, using Ladybugs 1 to 6: After placing Ladybugs 3 and 4, Ladybugs 5 and 6 can also be placed on the leaf.
 - For example, using Ladybugs 1 to 7: The first two consecutive numbers whose sum is greater than 7 are 4 and 5 so Ladybugs 4, 5, 6, and 7 can be placed on a single leaf.

Student Answers to Questions 1 to 3:

- 1. Now it's your turn. Start with Ladybugs 1 to 6. How many ladybugs can you place on the leaf without breaking the ladybug rule?
 - The most you can put on is 4.
 - Actual answers will vary, e.g., {1, 2, 4}; {1, 3, 5}; {1, 4, 6}; {3, 4, 5, 6}
- 2. What if you have Ladybugs 1 to 9 instead?
 - The most you can put on is 5.
 - Actual answers will vary, e.g., {1, 2, 4, 7}; {1, 3, 5, 7, 9}; {3, 4, 6, 8}; {5, 6, 7, 8, 9}
- 3. Now you have two leaves and Ladybugs 1 to 9. Each leaf needs to follow the ladybug rule. How many ladybugs can you place on the two leaves without breaking the ladybug rule?
 - a. You can place 6:
 - Answers will vary, e.g., {Ladybugs 1 & 2 on leaf 1; Ladybugs 3, 4, 5, & 6 on leaf 2}
 - b. You can place 7:
 - Answers will vary, e.g., {Ladybugs 1, 2, & 7 on leaf 1; Ladybugs 3, 4, 5, & 6 on leaf 2}
 - c. You can place 8:
 - {Ladybugs 1, 2, 4,and 8 on leaf 1. Ladybugs 3, 5, 6, and 7 on leaf 2}.
 - d. It is impossible to place 9.

Possible Student Answers to Challenge Questions 4 to 8:

4. You have two leaves and the ladybugs with odd numbers: Ladybugs 1, 3, 5, 7, and 9. How many of these ladybugs can you place on the two leaves without breaking the ladybug rule?

- All can land on the same leaf or in any order on two leaves.
- 5. Can you place Ladybugs 1, 3, 5, 7, 9, and 11? What if Ladybug 13 joins them? Ladybug 15? How many ladybugs with odd numbers can you place?
 - Infinite. They can all be on the same leaf or in any order on two leaves.
 - Any number of odd ladybugs can be placed on a single leaf because two odd numbers will always equal an even number: (2a+1)+(2b+1) = 2a+2b+2 = 2(a+b+1).
- 6. How many ladybugs with non-multiples of 3 can you place on two leaves?
 - Infinite. Place the ladybugs in alternating order on the leaves.
 - For non-multiples of 3, a similar odd-even principle holds. We alternate by leaf, all numbers on the left leaf have the form 3n+1 and on the right have the form 3n+2. But (3a+1)+(3b+1) = 3(a+b)+2 and (3a+2)+(3b+2) = 3(a+b+1)+1, so the sum of any two left-leaf numbers could only go on the right leaf and vice versa.
- 7. How many ladybugs with non-multiples of 4 can you place on two leaves?
 - Infinite. This is reduced to odds and evens as any number that leaves a remainder of 1 or 3 when divided by 4 goes on one leaf. Numbers that have a remainder of 2 go on the other leaf.

e.g., Leaf 1 will have 1, 3, 5, 7, 9, 11... and Leaf 2 will have 2, 6, 10, 14...

- 8. How many ladybugs with non-multiples of 5 can you place on two leaves?
 - Infinite. Numbers that leave a remainder of 1 or 4 when divided by 5 go on one leaf. Numbers that have a remainder of 2 or 3 go on the other leaf. e.g., Leaf 1 will have 1, 4, 6, 9... and Leaf 2 will have 2, 3, 7, 8...



Ladybugs Instructions

How many ladybugs can you place on the leaf without breaking the ladybug rule?

You'll be using number tiles for your ladybugs. Place tiles 1 to 6 above your leaf.



Ladybug Rule:

1. Two ladybugs cannot add up to a third ladybug on the leaf.



Can you place more than three ladybugs on the leaf without breaking the ladybug rule?

1. Now it's your turn. Start with Ladybugs 1 to 6. How many ladybugs can you place on the leaf without breaking the ladybug rule?



2. What if you have Ladybugs 1 to 9 instead?



3. Now you have **two leaves** and Ladybugs 1 to 9. Each leaf needs to follow the ladybug rule. How many ladybugs can you place on the two leaves without breaking the ladybug rule?



Ladybugs Challenge Tasks

4. You have two leaves and the ladybugs with odd numbers: Ladybugs 1, 3, 5,7, and 9. How many of these ladybugs can you place on the two leaves without breaking the ladybug rule?



Now you might want to switch to paper and pencil because some ladybugs will be larger than 9!

5. Can you place Ladybugs 1, 3, 5, 7, 9, and 11? What if Ladybug 13 joins them? Ladybug 15? How many ladybugs with odd numbers can you place?



6. How many ladybugs with non-multiples of 3 can you place on two leaves?



7. How many ladybugs with non-multiples of 4 can you place on two leaves?



8. How many ladybugs with non-multiples of 5 can you place on two leaves?



Julia Robinson Mathematics Festival











