KÖNIGSBERG
FESTIVAL GUIDE

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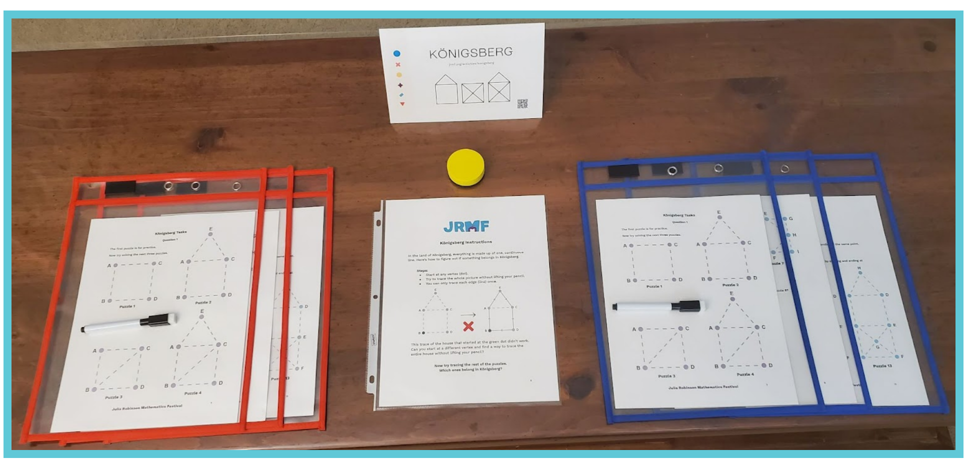
Julia Robinson Mathematics Festival

Materials and Setup

Per table (assuming 5 children per table), you will need:

Per Table	Material Preparation	
3 copies of Instructions	1-page sheet	p. 7
5 copies of Tasks	6-page sheet in dry erase sleeves *can be printed double-sided*	p. 8-13
1 copy of Table Sign	1-page sheet *print on cardstock for sturdiness*	p. 14
15 dry erase plastic sleeves		
5 dry erase markers		
5 dry erase marker erasers		

Per Table	Purchasing Materials		
dry erase combo	[30 piece set] for $22.53		Set comes with 30 plastic sleeves, 30 markers with erasers, and 4 extra erasers.
3 plastic sheet protectors	[pack of 100] for $7.67	[pack of 500] for $26.99	These are recommended in order to protect the instructions.



Objective

In the land of Königsberg, everything is made up of one, continuous line. Children will trace images to determine if something belongs in Königsberg.

Rules:
1. Start at any vertex (dot).
2. Try to trace the whole picture without lifting your pencil.
3. You can only trace each edge (line) once.

Materials

Each Königsberg table should be prepped for 5 stations.
Each station needs:
1. Königsberg instructions.
2. Königsberg tasks in dry erase sleeves.
3. 1 dry erase marker and eraser.

How to Play

We strongly encourage you to explore the activity yourself ahead of time.
You can try our digital version here: jrmf.org/puzzle/konigsberg

Introduce the activity without overexplaining it and without telling what strategies children might want to use. As much as possible, avoid giving away answers. Children should be encouraged to explore, experiment, and learn from their mistakes.

1. Tell children that Königsberg is an old puzzle, almost 300 years old. It began with an actual city named Königsberg that was crisscrossed by seven bridges. A famous mathematician named Leonhard Euler wanted to find a way to walk through the city so that he would cross each of those bridges once and only once. (He couldn't!)
2. Demonstrate the rules by tracing the first image (square) with them.
3. Ask the child to try tracing the second image (house). Encourage them to explain their thinking out loud as they trace. Have the child explore the rest of the tasks.

Standards

1. Make sense of problems and persevere in solving them. CCSS.MP1
2. Construct viable arguments and critique the reasoning of others. CCSS.MP3
3. Model with mathematics. CCSS.MP4
4. Look for and make use of structure. CCSS.MP7

Asking Good Questions

1. Ask questions about confidence.
    a. When a student asks you "Is this right?", instead of saying "yes" or "no" right away, ask them how confident they are in their answer. Here are some examples:
        i. "Maybe. What do you think? How confident are you?"
        ii. "On a scale of 1-5, how confident are you in your answer?"
    b. If a student is not confident in their answer, follow up by asking "What would help you feel more confident in your answer?" or "Why do you not feel confident?" This helps you determine how best to help the student through their explorations.

2. Ask students about choices.
    a. When a student is stuck or shows you a wrong answer, instead of jumping in and showing the student the correct answer, start by asking about the choices that the student made along the way. Here are some suggested steps to follow:
        i. Start from the beginning.
        ii. Ask students to show you what they've tried so far.
        iii. When the student gets to a point where they have different choices, ask the student "What other choices can you make here?"
        iv. Have the student make a different choice and try to solve the puzzle. This helps the student see that they have the power to make different choices during an activity, and they'll start to do this on their own in the future.
        v. If you're familiar with the puzzle or a particular solution, stop the student only when a different choice will help them get to the solution. This will help them feel successful faster without you giving away too much of the answer.

3. Ask students about strategies.
    a. If a student is getting into the activity and has been doing it for a while, ask the student if there are any strategies they've come up with to help them solve the puzzle or win the game.
    b. Follow up by asking if they think their strategies will work for all puzzles and/or larger puzzles, more complex puzzles, etc. Have the student explore more complex puzzles to test out their strategies.
    c. This is a great way to encourage a student to dive deeper into an activity and to start looking for patterns, structure, and proofs.

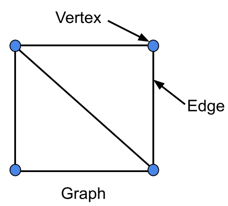
4. Activity specific questions.
    a. For each puzzle, circle your start and end points. What do you notice?
    b. Which puzzles were impossible? What do you think made them impossible?
    c. Can you predict whether a puzzle is possible before solving it? How do you know?

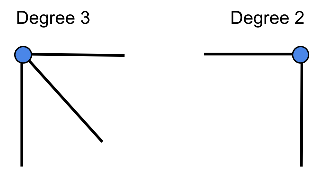
Answers

General Answers:

Note: This is for your information only. Although some children might notice that whether a puzzle is solvable is somehow related to the vertices, they are not expected to know/learn the language.

In school, the term “graph” usually refers to the *x* and *y* coordinates on a Cartesian plane. In graph theory, a graph is any set of points with lines connecting some of them. A point is called a vertex and a line connecting two vertices is called an edge. The number of edges connected to a vertex is called the degree of that vertex.





1. If an image has no vertices with an odd degree (i.e. a point with an odd number of lines connected to it), it is possible to trace starting at any vertex.
2. If an image has two vertices with an odd degree, it is only possible to trace starting at one of these two vertices.
3. If an image has any other number of vertices with an odd degree, it is impossible to trace.

Answers to Questions 1 to 9:

1. Solve puzzles 1, 2, 3, 4
   - Puzzle 1 is solvable starting at any point and will always end back at the starting point.
   - Puzzle 2 is solvable starting at A or C.
   - Puzzle 3 is solvable starting at B or C.
   - Puzzle 4 is solvable starting at A or B.

2. One of these puzzles is possible and the other is impossible. Which puzzle is possible? Write out how you would solve it. Which puzzle is impossible? Can you find a path that traces as much of the picture as possible?
   - Puzzle 5 is possible starting at B or E.
   - Puzzle 6 is impossible, e.g., The closest I got on Puzzle 6 is: C to D to G to C to B to A to F to E to D.

3a. Derrick solved puzzle 7 by starting at c and ending at c. Can you solve puzzle 7 starting at a different point? How many different points can you start at and still solve puzzle 7?
   - Solvable from any point.

3b. Derrick solved puzzle 8 by starting at a and ending at g. Can you solve puzzle 8 starting at a different point? How many different points can you start at and still solve puzzle 8?
   - Puzzle 8 is solvable starting at A or G.

4. Try puzzles 9 and 10. For each puzzle, how many different points can you start at and solve the puzzle?
    - Puzzle 9 is solvable from any starting point.
    - Puzzle 10 is impossible because it has more than 2 odd-degree vertices.
5. Try puzzles 11, 12, and 13. You can solve one of these puzzles by starting and ending at the same point. Which puzzle is it? What makes this puzzle special? Why do you think you can't solve the other puzzles by starting and ending at the same point?
    - Puzzle 11 is solvable starting at A or D.
    - Puzzle 12 is solvable starting and ending at the same point because it doesn't have any odd-degree vertices.
    - Puzzle 13 is impossible because it has more than 2 odd-degree vertices.
6. Erin made 2 three-story towers that aren't in the app. One of these two towers is impossible to trace. Make a prediction: Which tower do you think is impossible to trace? Why?
    - Tower 1 is solvable starting at A or D.
    - Tower 2 is impossible because it has more than 2 odd-degree vertices.

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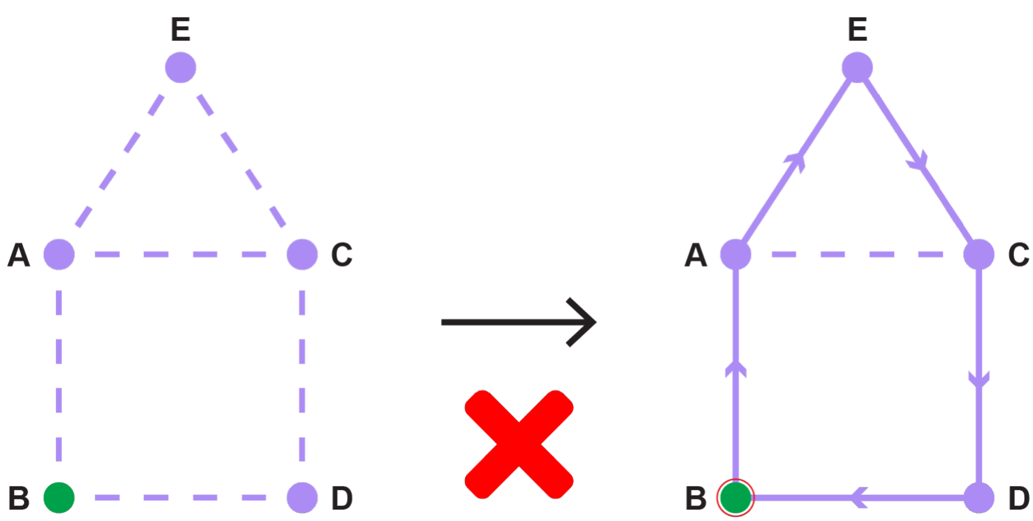
Königsberg Instructions

In the land of Königsberg, everything is made up of one, continuous line. Here's how to figure out if something belongs in Königsberg.

Steps:

- Start at any vertex (dot).
- Try to trace the whole picture without lifting your pencil.
- You can only trace each edge (line) once.



This trace of the house that started at the green dot didn't work. Can you start at a different vertex and find a way to trace the entire house without lifting your pencil?

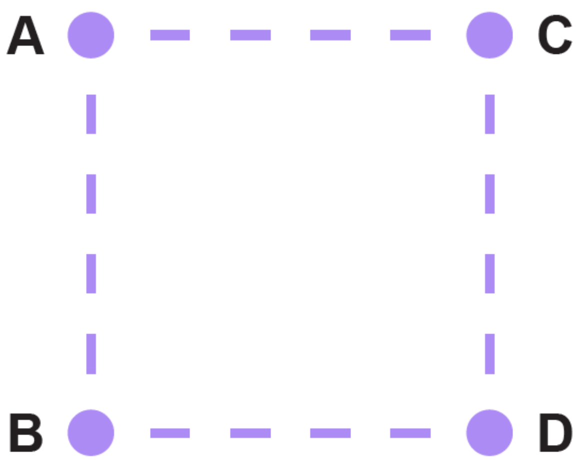
Now try tracing the rest of the puzzles.
Which ones belong in Königsberg?

Königsberg Tasks

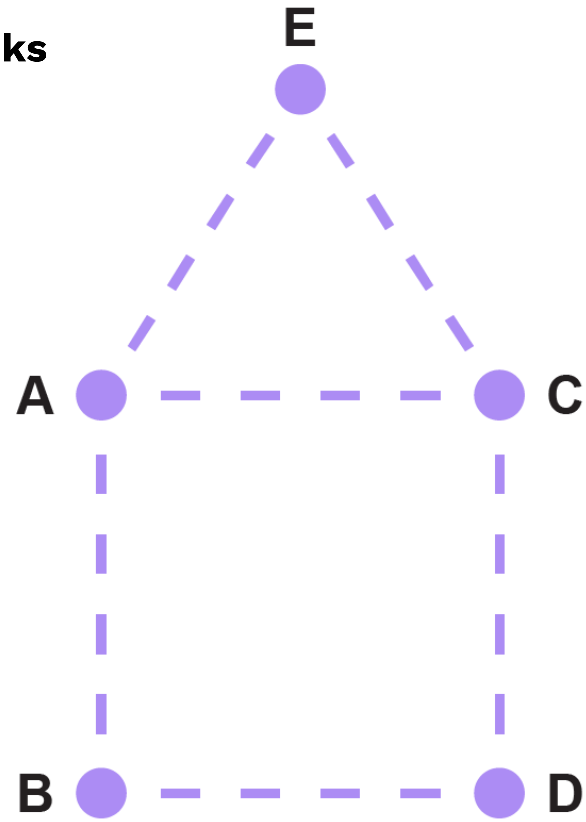
Question 1

The first puzzle is for practice.

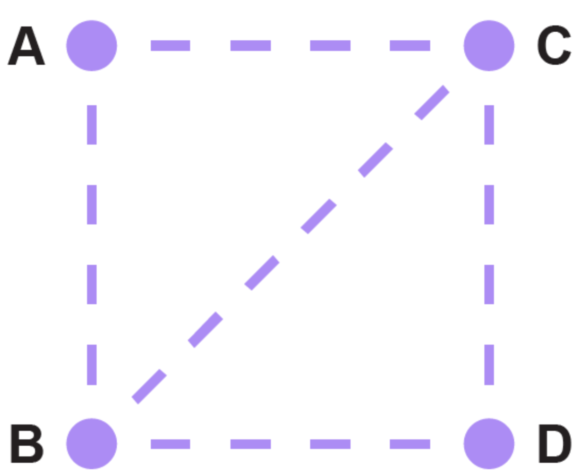
Now try solving the next three puzzles.



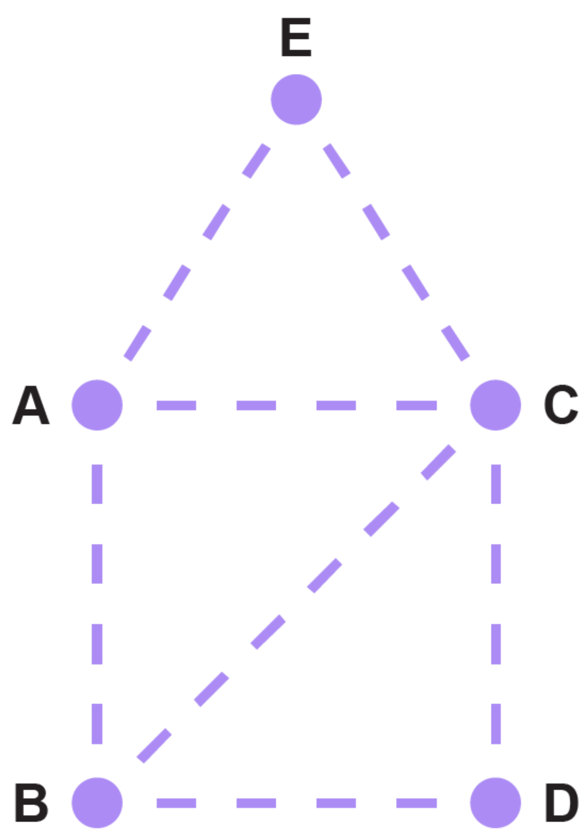
Puzzle 1



Puzzle 2



Puzzle 3



Puzzle 4

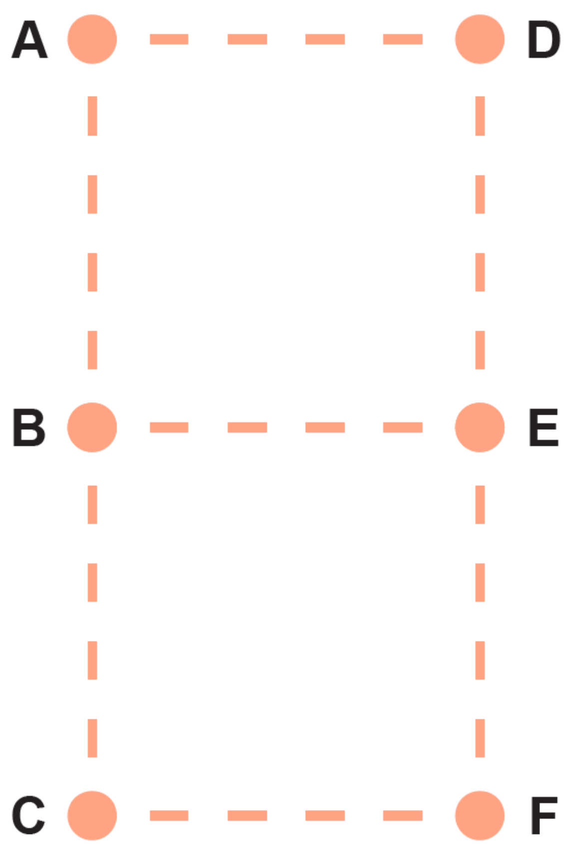
Question 2

One of these puzzles is possible and the other is impossible.

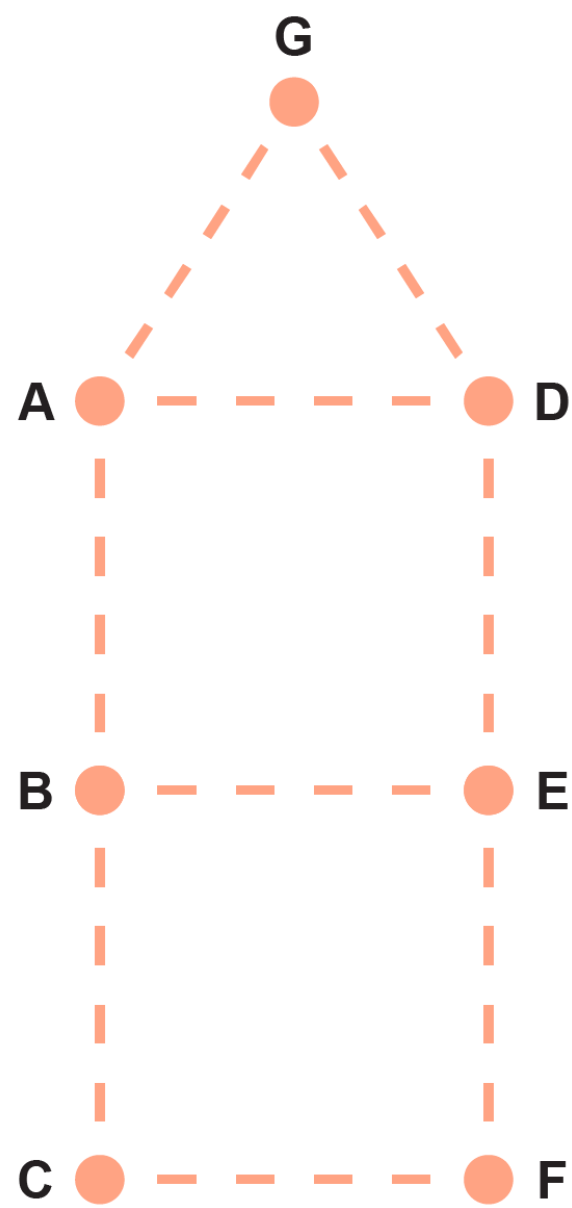
Which puzzle is possible?

Which puzzle is impossible?

Can you find a path that traces as much of the picture as possible?



Puzzle 5



Puzzle 6

Question 3a

Derrick solved puzzle 7

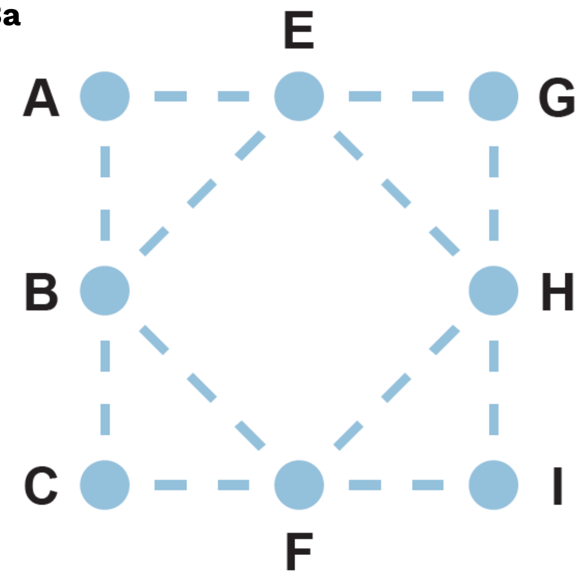
by starting at c and ending at c.

Can you solve puzzle 7 starting

at a different point?

How many different points can you

start at and still solve puzzle 7?



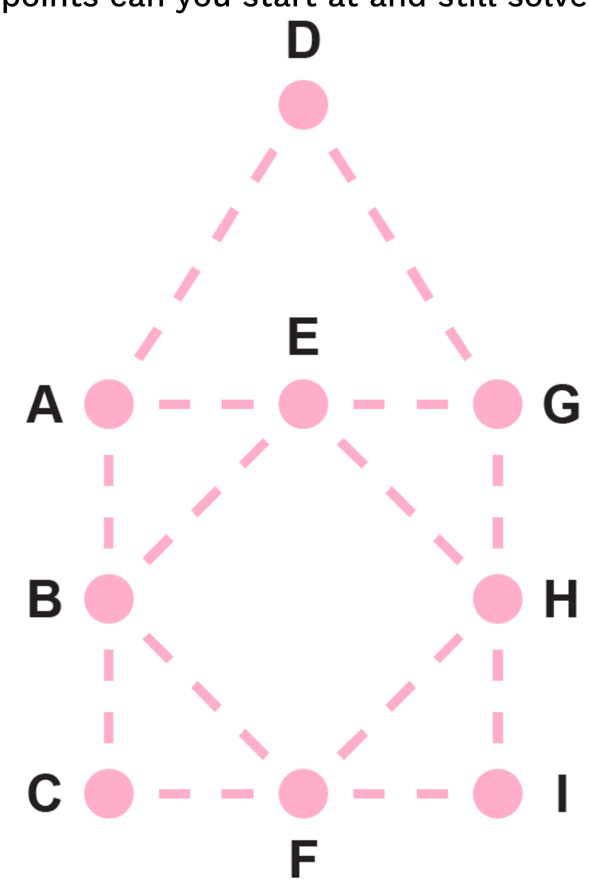
Puzzle 7

Question 3b

Derrick solved puzzle 8 by starting at a and ending at g.

Can you solve puzzle 8 starting at a different point?

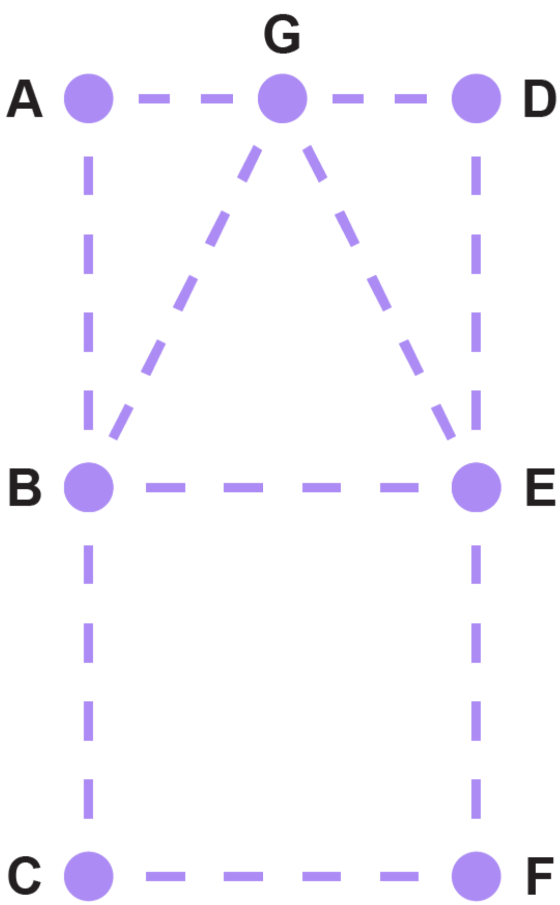
How many different points can you start at and still solve puzzle 8?



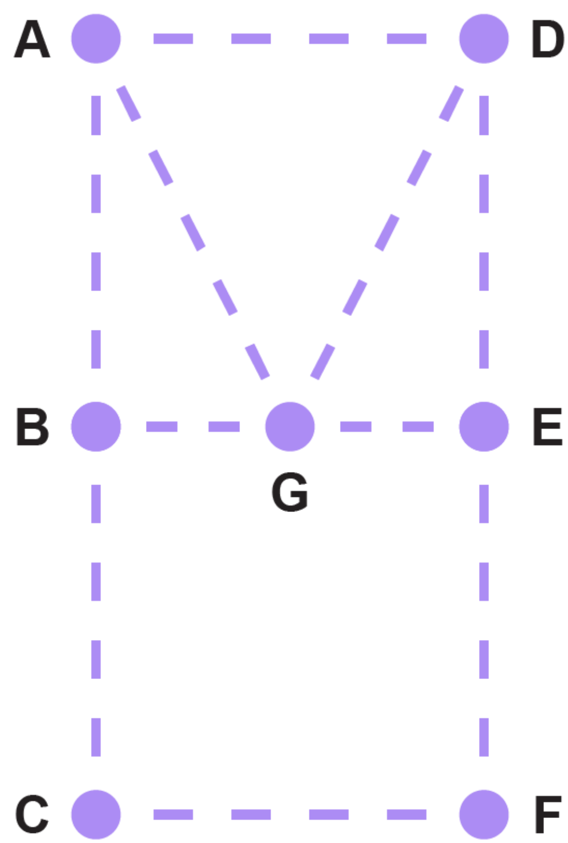
Question 4

Try puzzles 9 and 10.

For each puzzle, how many different points can you start at and solve the puzzle?



Puzzle 9



Puzzle 10

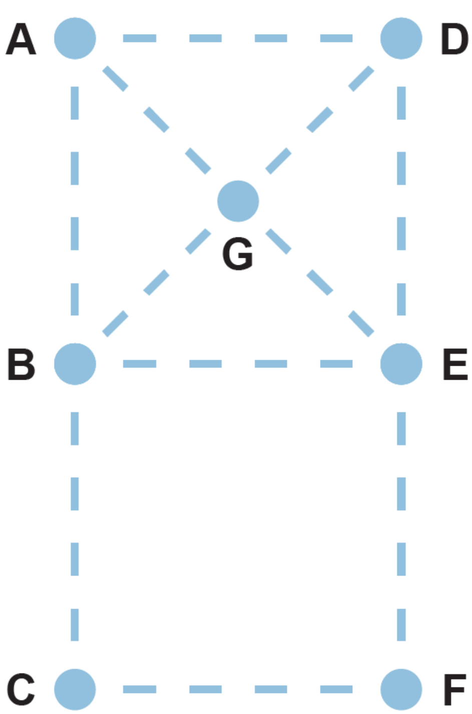
Question 5

Try puzzles 11, 12, and 13.

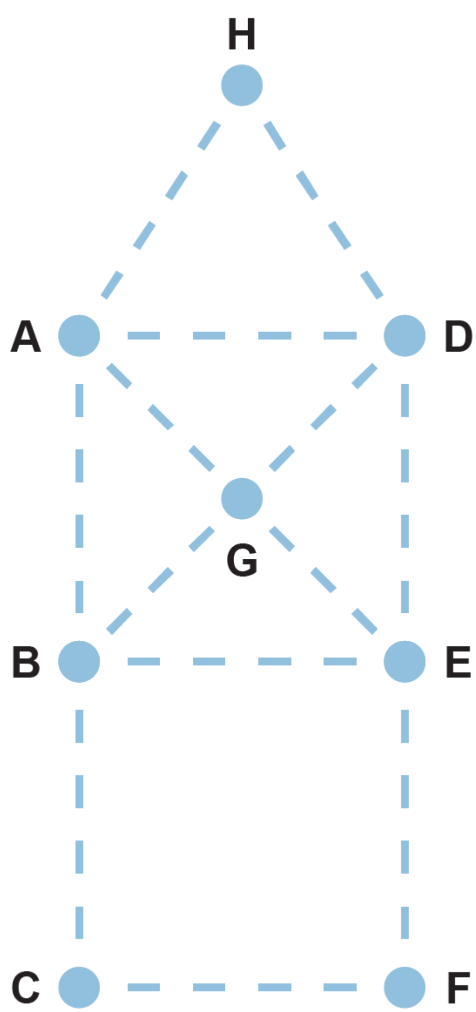
You can solve one of these puzzles by starting and ending at the same point. Which puzzle is it?

What makes this puzzle special?

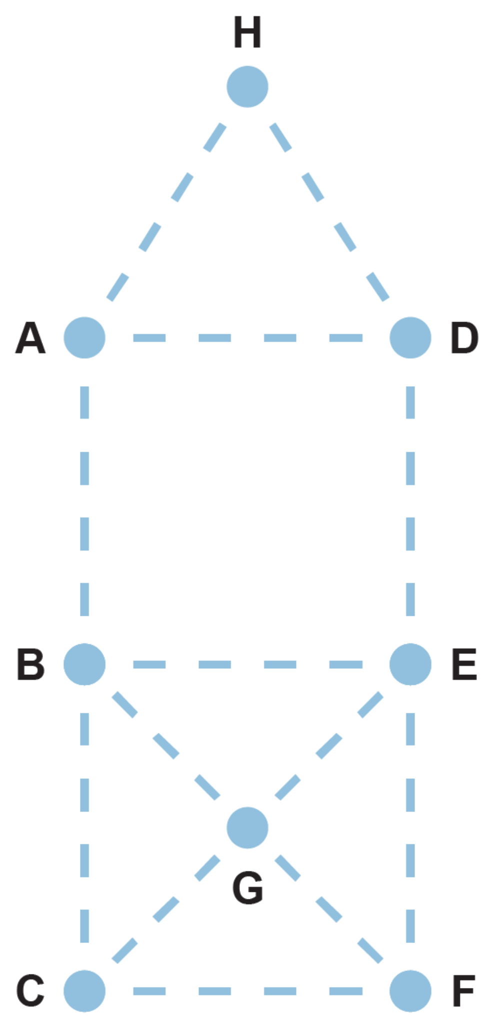
Why do you think you can't solve the other puzzles by starting and ending at the same point?



Puzzle 11



Puzzle 12

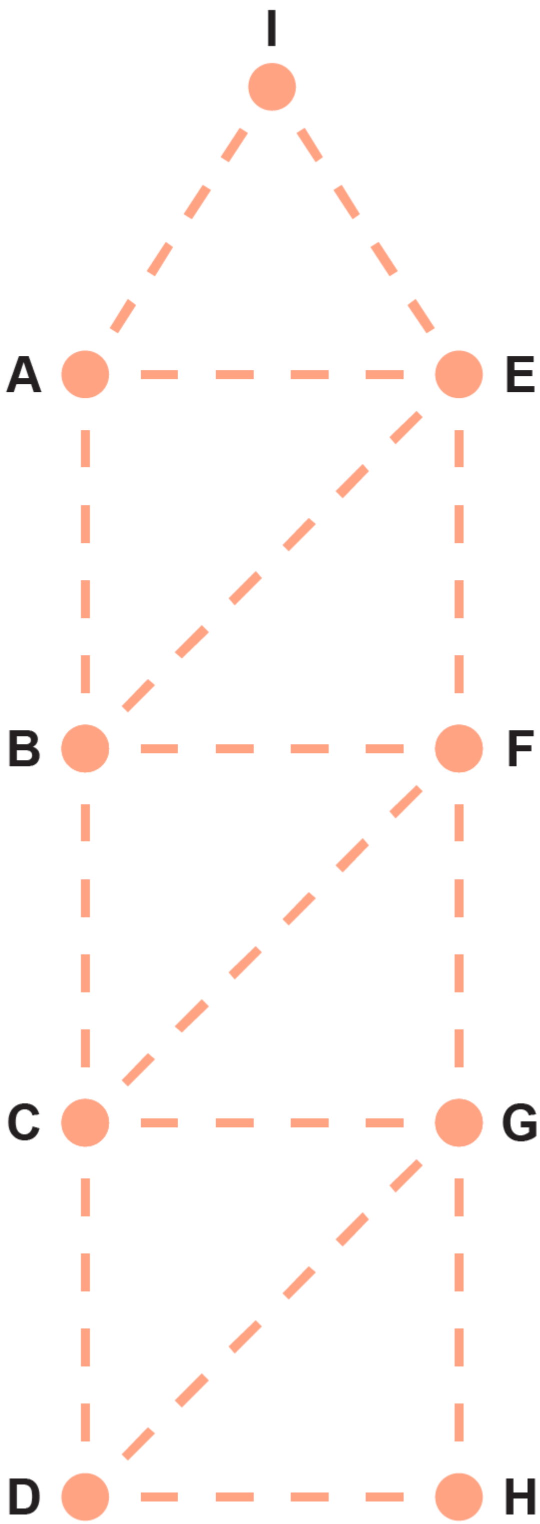


Puzzle 13

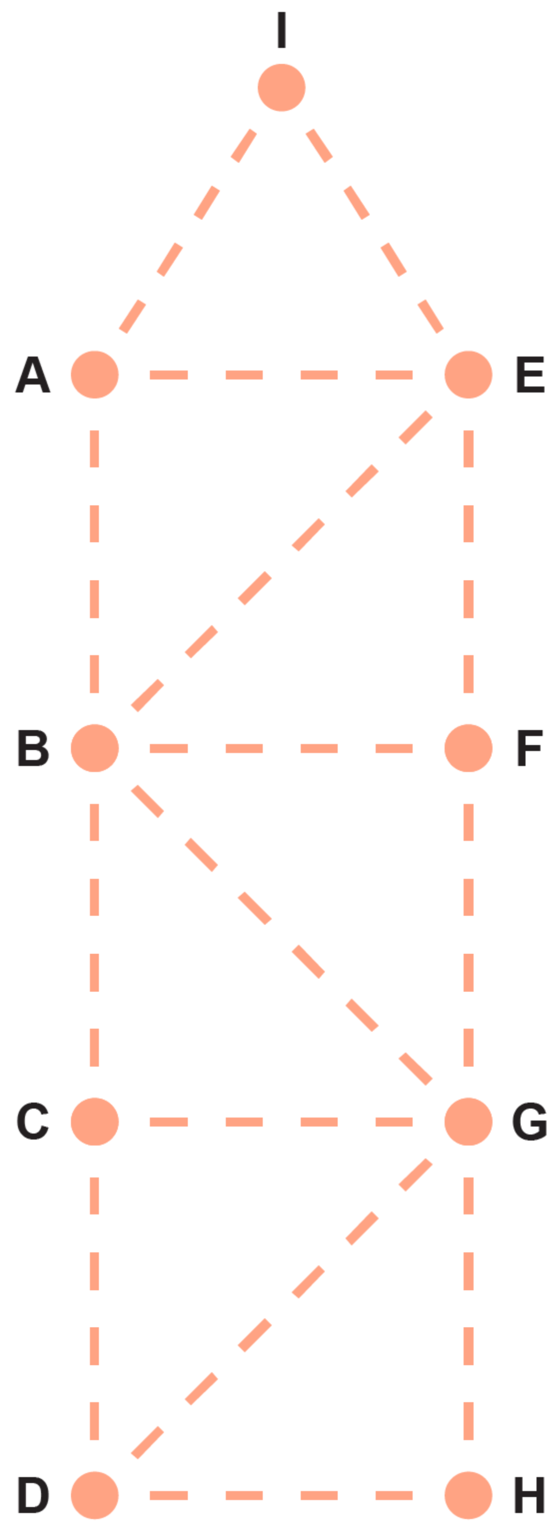
Question 6

One of these three-story towers is impossible to trace.

Make a prediction: Which tower do you think is impossible to trace? Why?



Tower 1



Tower 2

KÖNIGSBERG

Play for free at
jrmf.org/puzzle/konigsberg

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