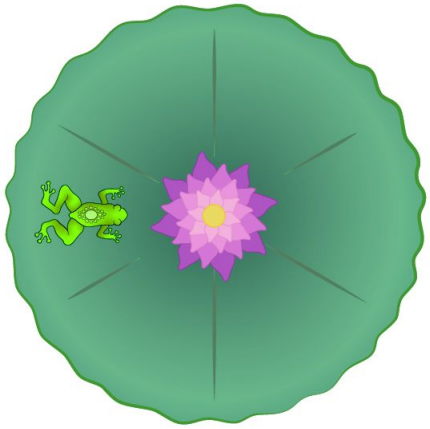
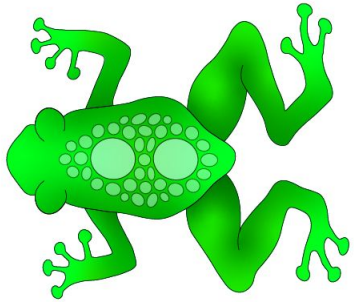
Tippy Toads





Julia Robinson Mathematics Festival

San Francisco Math Circle at San Francisco State University

App

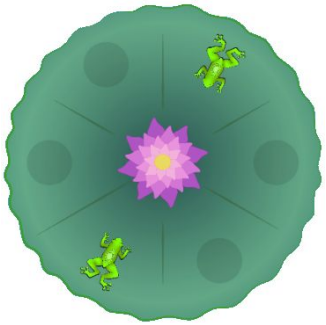
jrmf.org

Tippy Toads

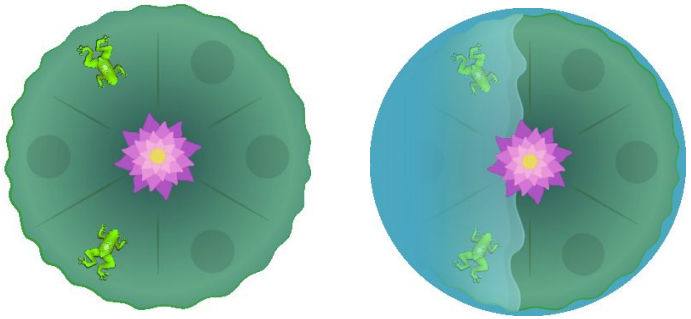
Objective:
Some toads have found a lilypad to sunbathe on! If they don't spread themselves nicely around the edge, the lilypad will tilt and some toads will end up underwater. Which numbers of toads can balance?

Rules:

- Each of the six spaces on the lilypad can seat a toad.
- When the toads are balanced, they can sunbathe in comfort! (What do you think "balanced" might mean?)
- If the toads are unbalanced, some will end up underwater.



A balanced arrangement



An unbalanced arrangement

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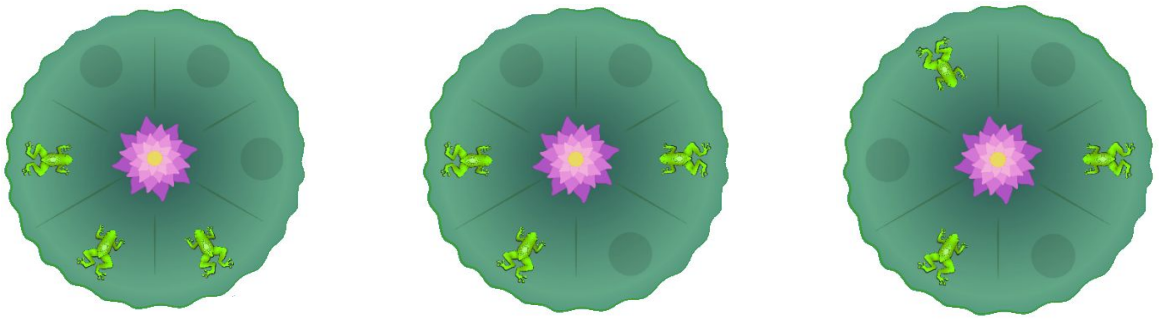
Revised on 2021.11.08

Six Spaces

1. The previous slide shows a way to balance two toads. Can you find a way to balance three toads?
2. Is there a way to predict which of the arrangements below would balance just by looking at them?



3. Can a single toad be balanced?
4. Can you find a way to balance four toads? Five? Six?
5. Were there any numbers you weren't able to balance? Can you explain why there is no way to balance them without having to try every possible arrangement?

Seven or Eight Spaces

1. With seven spaces, which numbers of toads can be balanced on the lilypad? Keep track of your findings.
2. With eight spaces, which numbers of toads can be balanced on the lilypad? Keep track of your findings.
3. You've probably noticed that things work out pretty differently for seven and eight spaces! Why is that? Could you have predicted they would be so different?

More Spaces

1. See if you can use the patterns you discovered with six, seven, and eight spaces to predict which numbers will balance for nine spaces and check to see if you were right.
2. Can you predict what will happen with ten spaces? Eleven spaces?
3. Do your predictions still work with twelve spaces or has something changed? (Be careful!)
4. When confronted with each new number of spaces, what are some strategies you can use to find balanced arrangements?
5. Try even larger numbers of spaces. Does your strategy still work? If so, can you explain why it works? If not, can you find a way to fix it?

Large Toads

1. Instead of just small toads, suppose we also have large toads that weigh twice as much. If we allow for each space to be occupied by a small toad, a large toad, or no toad, how does this change the numbers of toads we can balance on a lilypad?
2. Try revisiting a lilypad with six spaces. Are there any numbers of toads that you can balance now that you couldn't balance with small toads alone?
3. Revisit seven to twelve spaces. What new numbers of toads can you balance for each? Do any of your strategies for small toads help with both small and large toads?
4. Is there a general pattern that helps predict which numbers of toads can be balanced on which numbers of spaces?