

Pool Testing



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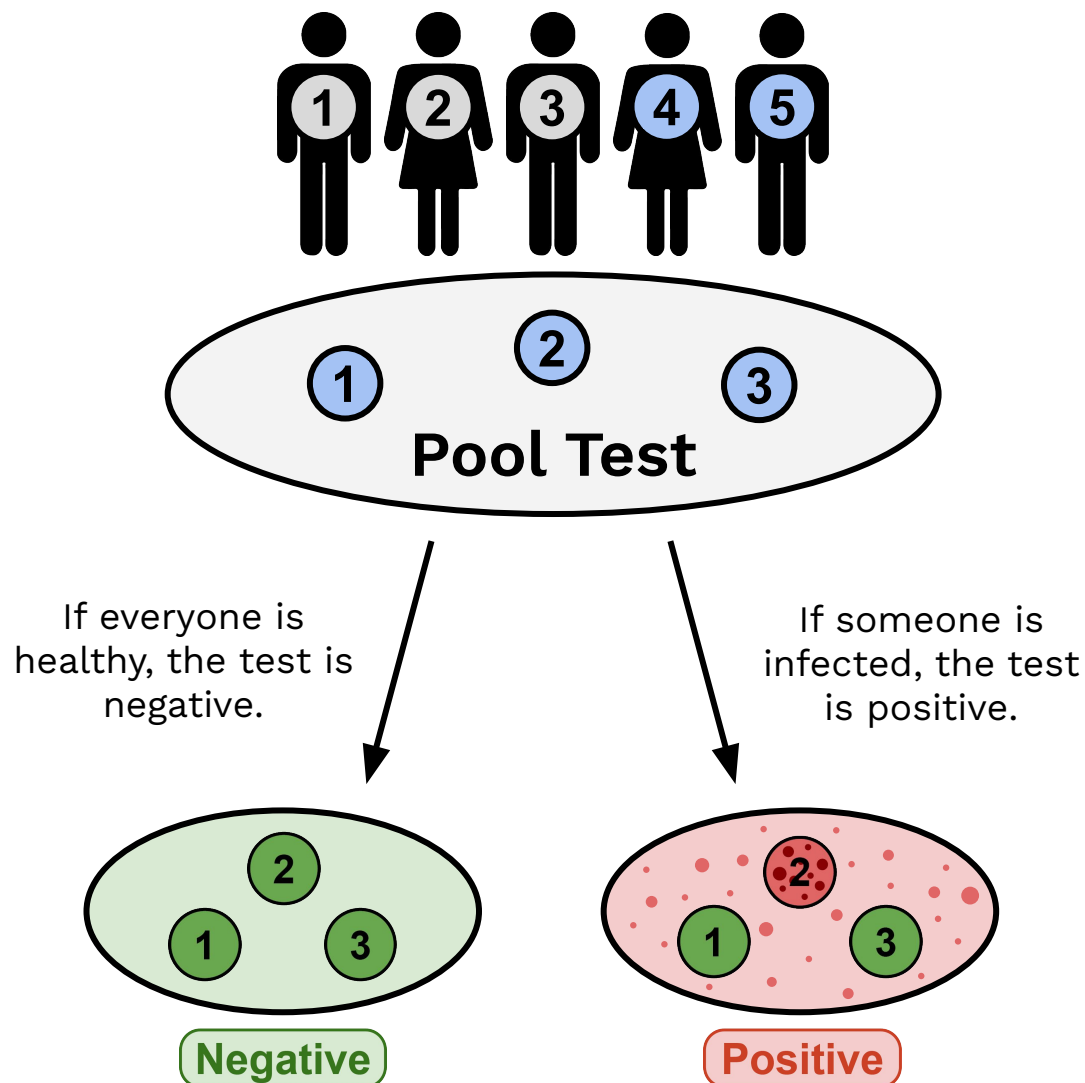


App

jrmf.org

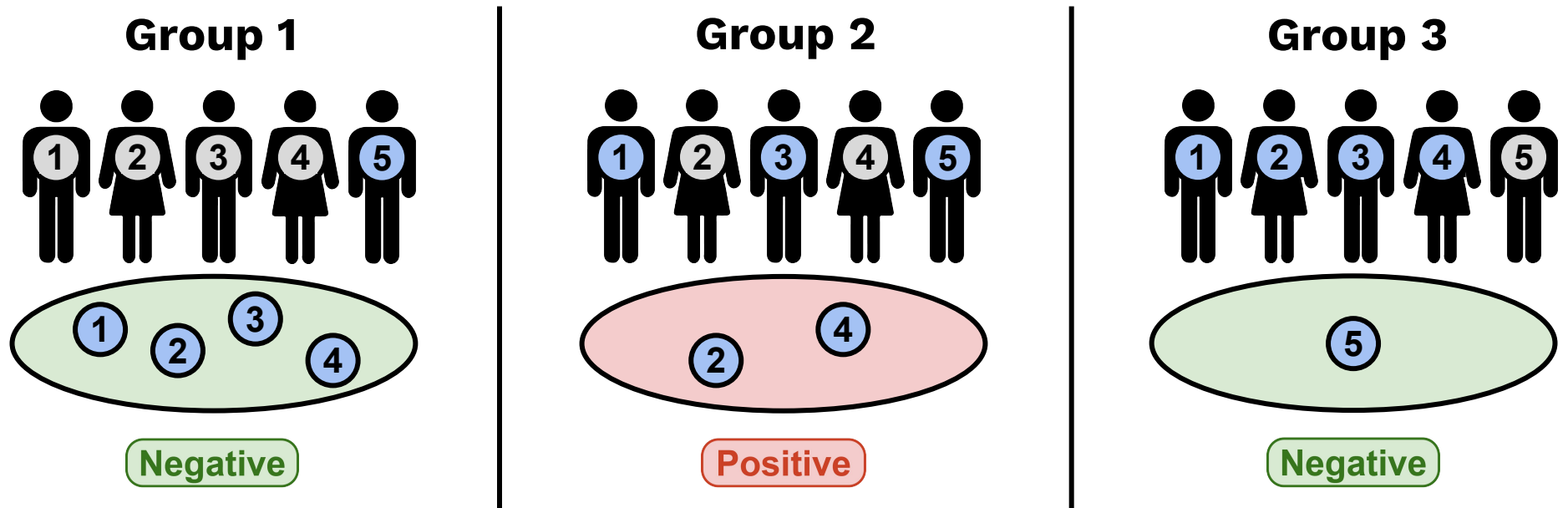
What is pool testing?

Suppose someone is infected with a virus. If there aren't enough tests to test each person individually, a pool test can be used to test many people at the same time. In this example, three people are tested at the same time.



Positive or Negative

- If everyone in a pool test is healthy, the pool test is negative.
- If the infected person is in a pool test, the pool test is positive.
- A pool test doesn't usually tell you who is infected, but it does tell you who might be infected.
- Three *different* groups of people were tested below. For each group, can you figure out who the infected person is? If not, can you figure out all of the people who might be infected?

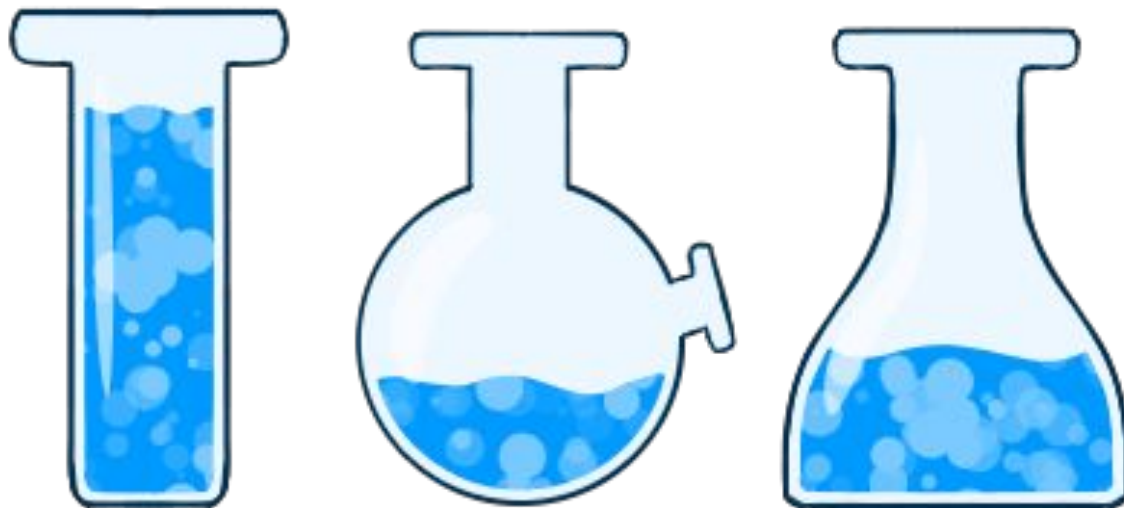


Using Pool Testing

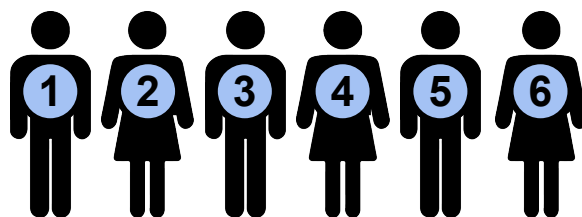
Goal: Find the infected patient using as few pool tests as possible.

Rules:

- Exactly one patient is infected.
- You can do pool tests of any size (including a pool of just one person).
- You will usually need to do more than one pool test to find the infected person. You can test the same patient any number of times.



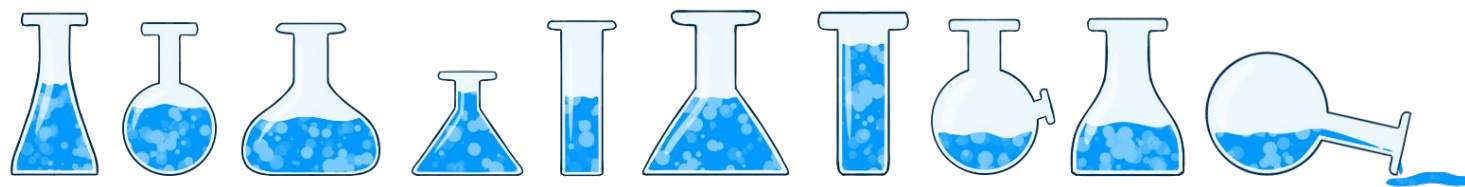
1. There are 6 patients and exactly 1 patient is infected. What is the fewest number of tests you need to guarantee that you will always be able to find the infected patient?
2. What if there are 7 patients? 8 patients? 9?
3. If you are only able to do 5 tests, what is the largest number of patients you can test so that you can always find the infected one?



Who would *you* test first?



1. One day, you only have enough time to do one round of pool testing. You can do as many pool tests as you want, but you must do all of the tests at once. You cannot use the result of one pool test to decide how you'll do another pool test.
 - a. If there are 6 patients and exactly 1 patient is infected, what is the fewest number of pool tests you need to guarantee that you will always be able to find the infected person.
 - b. What if there are 7 patients? 8 patients? 9?
 - c. If you are only able to do 5 tests, what is the largest number of patients you can test so that you can always find the infected one?
2. The clumsy lab technician often spills one of the tests. However, you don't know which test result is going to be lost. Knowing this, how should you do your pool testing to guarantee that you'll find the infected person?



A Spreading Infection

1. There are 6 patients and exactly 2 patients are infected. What is the fewest number of tests you need to guarantee that you will always be able to find both of the infected patients?
2. What if there are 7 patients? 8 patients? 9?
3. If you are only able to do 5 tests, what is the largest number of patients you can test so that you can always find both of the infected ones?
4. What if exactly 3 patients are infected? 4 patients? 5? More?
5. What if there is an unknown number of patients infected?

