Crack the Code

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San Francisco Math Circle at SAN FRANCISCO STATE UNIVERSITY
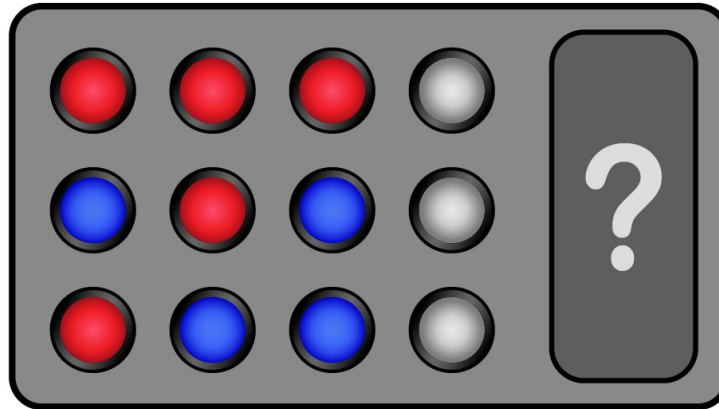
App

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Objective:
You are a hacker who has discovered a security flaw in a top secret mainframe. Crack the secret code in as few guesses as you can!

Rules:
- Each turn, you guess a code by placing three lights, each of which can be red or blue.
- If at any point you guess all colors correctly in the right order, you win!
- If you don't guess the code exactly, the machine will let you know how many colors are in the right positions, which you can use to improve your next guess.
- If you don't guess the code in four tries, you lose.
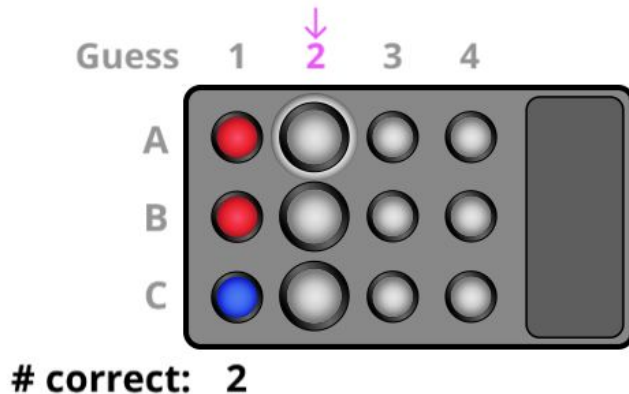
The board to the left indicates that the first guess has two colors in the right position, but we don't know which ones!

1. Play a few games with 2 colors and 3 lights. How often are you able to win?

2. What information does guessing all red lights or all blue lights tell you? Is guessing both of these helpful?

3. Are some codes easier to crack than others or do you think they are all equally difficult?

4. Is it always possible to crack the code in four guesses? If so, find a strategy for always cracking the code in four guesses and explain it to a friend.

5. Is there a "best" first guess or could you make a similar strategy starting from *any* first guess?

1. Play a few games with the puzzle set to 4 lights and 5 guesses.

2. Is it always possible to crack the code in 5 guesses? If so find a strategy for always cracking the code in 5 guesses and explain it to a friend.

3. Now try with the puzzle set to 5 lights and 6 guesses. Can you always crack the code? What's your strategy?

4. Is it always possible to solve the n-light puzzle in $n+1$ guesses? If your strategies for each version have been different, see if you can find a single strategy that works for 3 lights with 4 guesses, 4 lights with 5 guesses, and 5 lights with 6 guesses.
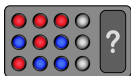
3-Color Codes

1. Play a few games with 3 colors, 3 lights, and 5 guesses.

2. If you make a first guess and change only one light's color for your second guess, what information do you learn if the number of correct positions goes up? Goes down? Stays the same?

3. Can you find a strategy to always crack the code in 5 guesses?

4. How many guesses do you think you need to always crack a 3-color, 4-light code? Set the puzzle to 4 lights and that many guesses and see if you can find a strategy.

5. If you were able to do it in 6 guesses, can you find a general strategy for solving an n-light code in $n+2$ guesses? If not, is there something else you can say?

2-Color Challenges

1. If you have a strategy for always cracking the 2-color, 3-light code in 4 guesses, you might wonder if you could always crack it in 3 guesses. Can you? Or can you see a reason why it's impossible?

2. Is it possible to always crack a 4-light code in 4 guesses? Why or why not? (Hint: Regardless of your first guess, how many codes share exactly two positions with it? What could you do to tell those codes apart in 3 guesses?)

3. They're *very* tricky to find, but there are strategies for always cracking a 5-light code in 5 guesses! Can you find such a strategy?

4. What does always being able to crack a 5-light code in 5 guesses tell you about *n*-light codes?

5. Is it ever the case that all *n*-code codes can be cracked in *fewer* than *n* guesses?

1. Is it possible to always crack a 3-color, 3-light code in 4 guesses? If so, find the strategy. If not, explain why not.

2. They're *very* tricky to find, but there are strategies for always cracking a 3-color, 4-light code in 5 guesses! Can you find such a strategy? Does this tell you anything about 5-light puzzles?

3. How many guesses do you need to always crack a 4-color, 3-light code?