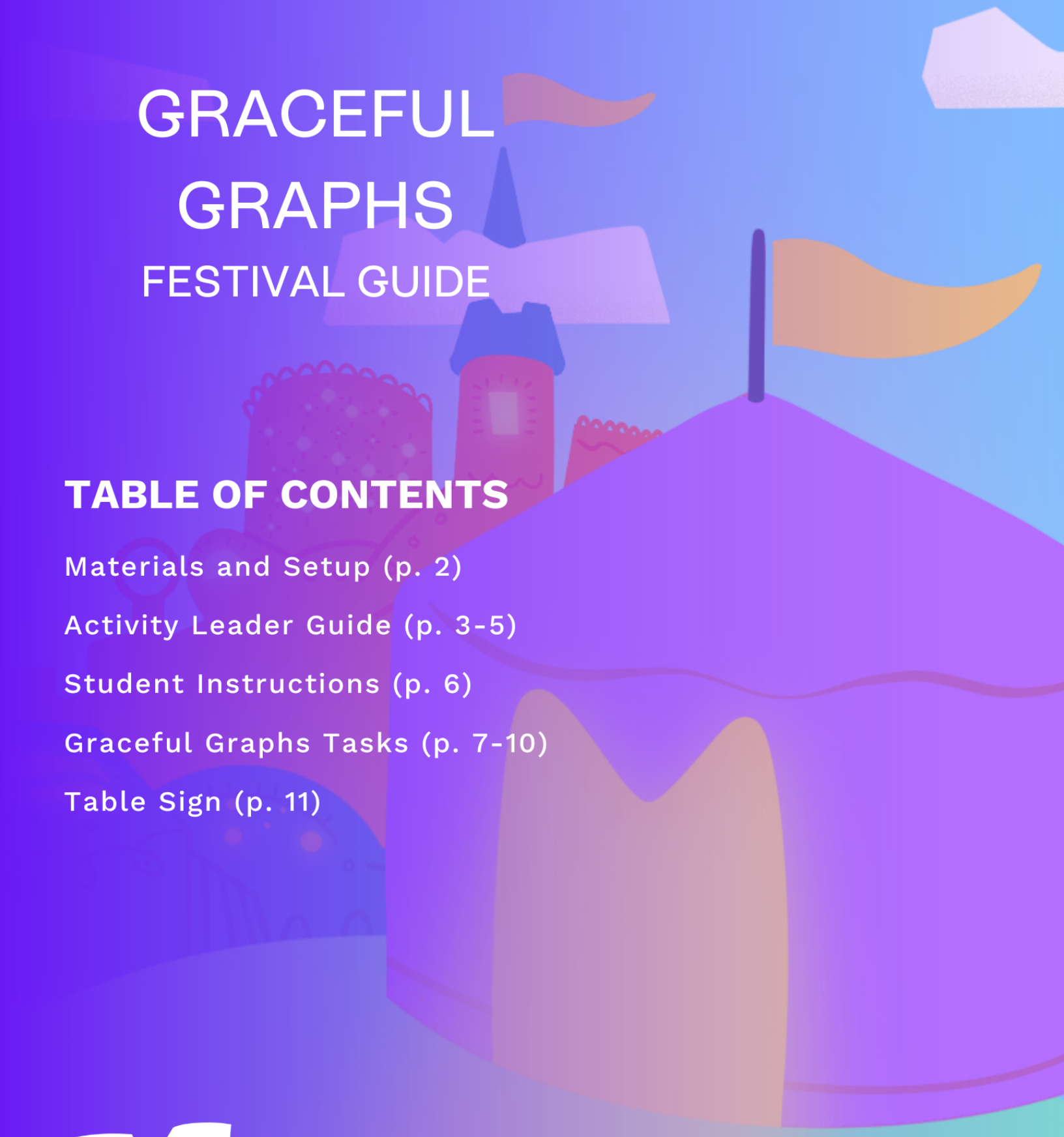
GRACEFUL GRAPHS

FESTIVAL GUIDE



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Materials and Setup

Per table (assuming 5 students per table), you will need:

Per Table	Material Preparation	
3 copies of Instructions	1-page sheet	p. 6
5 copies of Tasks	1-page sheet *can be printed double-sided*	p. 7-10
1 copy of Table Sign	1-page sheet *print on cardstock for sturdiness*	p. 11
5 dry erase plastic sleeves		
5 dry erase markers		
5 dry erase marker erasers		

Per Table	Purchasing Materials		
dry erase combo	30 piece set for $22.53		Set comes with 30 plastic sleeves, 30 markers, and 4 erasers.
dry erase markers	pack of 72 for 9.99		If you need just the markers.
3 plastic sheet protectors	pack of 100 for $7.67	pack of 500 for $26.99	These are recommended in order to protect the instructions.

Objective

Place numbers in the circles to gracefully label each graph.

Rules:
1. Place the numbers in the number list into the circles.
2. You can use each number only once.
3. For each line, find the difference between the two numbers it connects. A graph is graceful if every difference is different.

Materials

Each Graceful Graph table should be prepped for 5 stations.
Each station needs:
1. Graceful Graph instructions.
2. Graceful Graph tasks in dry erase plastic sleeves.
3. 1 dry erase marker and eraser.

How to Play

We strongly encourage you to explore the activity yourself ahead of time.
You can try our digital version here: jrmf.org/puzzle/graceful-gourds

Introduce the activity without overexplaining it and without telling what strategies students might want to use. As much as possible, avoid giving away answers. Students should be encouraged to explore, experiment, and learn from their mistakes.

1. Ask the student to write the numbers 0, 1, 2, and 3 in the circles of the first challenge.
2. Ask them to write the difference of the pairs of numbers on the line connecting them.
3. Point out the line numbers. Ask if they can rearrange the numbers in the circles so that the line numbers are all different, making a ***graceful graph***.
4. Have the student rearrange the 4 numbers to make a graceful graph.

Standards

1. Make sense of problems and persevere in solving them. CCSS.MATH.PRACTICE.MP1
2. Construct viable arguments and critique the reasoning of others. CCSS.MATH.PRACTICE.MP3
3. Model with mathematics. CCSS.MATH.PRACTICE.MP4
4. Look for and make use of structure. CCSS.MATH.PRACTICE.MP7

Asking Good Questions

1. Ask questions about confidence.
    a. When a student asks you “Is this right?”, instead of saying “yes” or “no” right away, ask them how confident they are in their answer. Here are some examples:
        i. “Maybe. What do you think? How confident are you?”
        ii. “On a scale of 1-5, how confident are you in your answer?”
    b. If a student is not confident in their answer, follow up by asking “What would help you feel more confident in your answer?” or “Why do you not feel confident?” This helps you determine how best to help the student through their explorations.

2. Ask students about choices.
    a. When a student is stuck or shows you a wrong answer, instead of jumping in and showing the student the correct answer, start by asking about the choices that the student made along the way. Here are some suggested steps to follow:
        i. Start from the beginning.
        ii. Ask students to show you what they’ve tried so far.
        iii. When the student gets to a point where they have different choices, ask the student “What other choices can you make here?”
        iv. Have the student make a different choice and try to solve the puzzle. This helps the student see that they have the power to make different choices during an activity, and they’ll start to do this on their own in the future.
        v. If you’re familiar with the puzzle or a particular solution, stop the student only when a different choice will help them get to the solution. This will help them feel successful faster without you giving away too much of the answer.

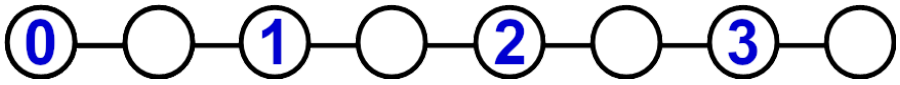
3. Ask students about strategies.
    a. If a student is getting into the activity and has been doing it for a while, ask the student if there are any strategies they’ve come up with to help them solve the puzzle or win the game.
    b. Follow up by asking if they think their strategies will work for all puzzles and/or larger puzzles, more complex puzzles, etc. Have the student explore more complex puzzles to test out their strategies.
    c. This is a great way to encourage a student to dive deeper into an activity and to start looking for patterns, structure, and proofs.

4. Activity specific questions.
    a. Are there any numbers that have to be placed next to one another?

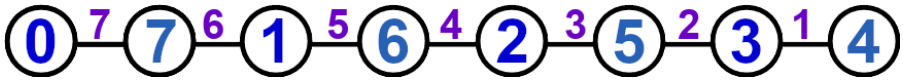
Answers

1. If a graph has *m* edges then the number *m* must appear as an edge label. Since edge labels are the difference between two vertices, the numbers 0 and *m* must appear on adjacent vertices because *m-0* is the only difference yielding *m*.

2. All paths are graceful.
   a. One way to gracefully label a path is to start with 0 at the left and then placing increasing numbers moving left-to-right, skipping every other vertex.



   Once you have reached the end, continue placing numbers in increasing order, moving right-to-left starting from the rightmost blank vertex:



   This provides a solution that gives an edge labeling in decreasing order, left-to-right

   b. The symmetry of a path allows for any graceful labeling to also be carried out in reverse.

3. The number of graceful labelings for 1- to 5-paths are as follows:

m edges	Graceful Labelings
1	2
2	4
3	4
4	8
5	24

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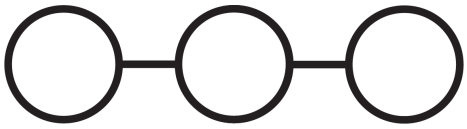
Graceful Graphs Instructions

Can you place the numbers in the circles and make each graph graceful?

Number List: 0, 1, 2



Rules:

1. Place the numbers in the number list into the circles.
2. You can use each number only once.
3. For each line, find the difference between the two numbers it connects. A graph is **graceful** if every difference is different.

Number List: 0, 1, 2

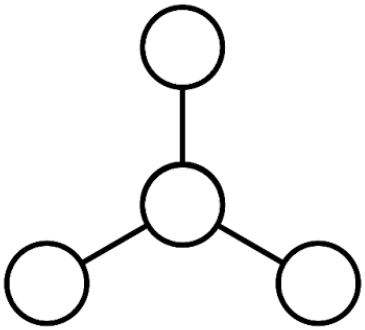
1-0=1 2-1=1

Number List: 0, 1, 2

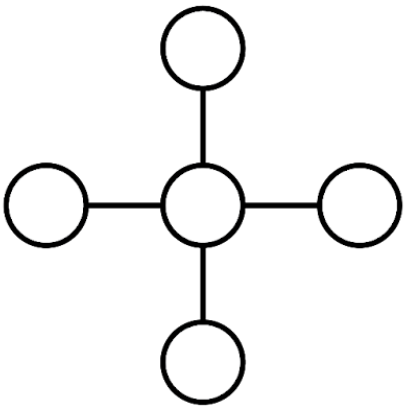
2-0=2 2-1=1

Graceful Graphs Challenge #1: Stars

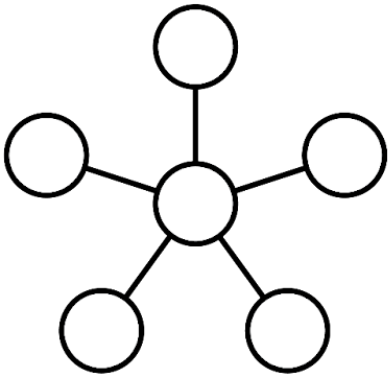
Number List:
0, 1, 2, 3



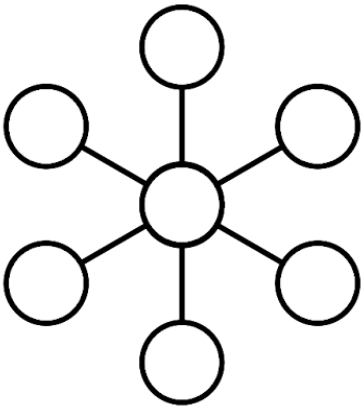
Number List:
0, 1, 2, 3, 4



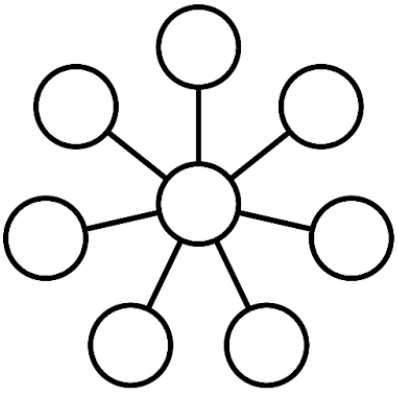
Number List:
0, 1, 2, 3, 4, 5



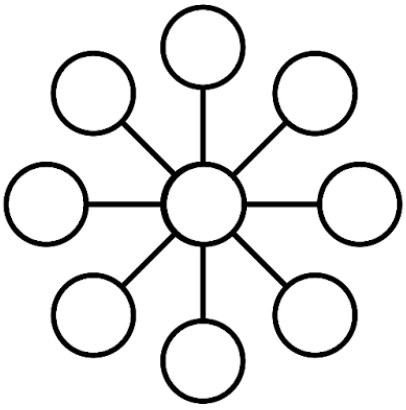
Number List:
0, 1, 2, 3, 4, 5, 6



Number List:
0, 1, 2, 3, 4, 5, 6, 7



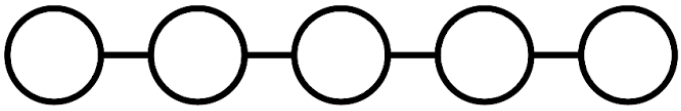
Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8



Graceful Graphs Challenge #2: Paths

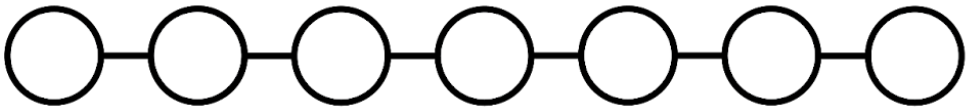
Number List: 0, 1, 2, 3

Number List: 0, 1, 2, 3, 4

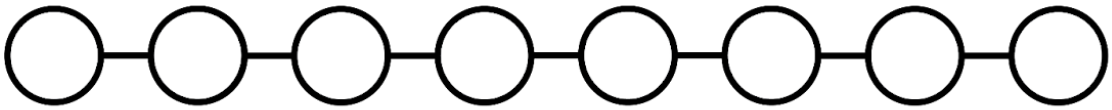


Number List: 0, 1, 2, 3, 4, 5

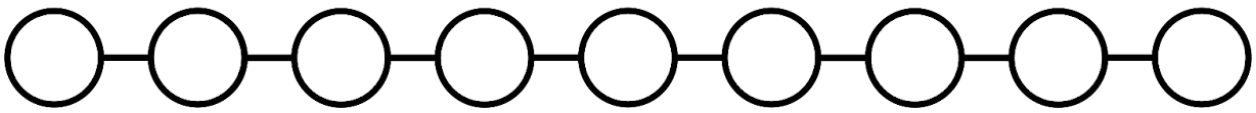
Number List: 0, 1, 2, 3, 4, 5, 6



Number List: 0, 1, 2, 3, 4, 5, 6, 7

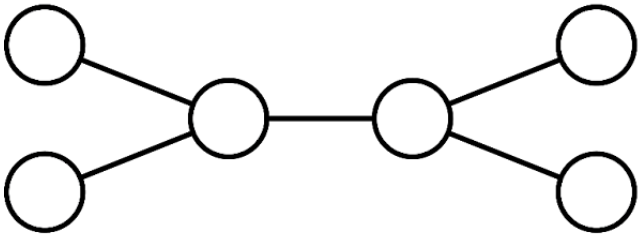


Number List: 0, 1, 2, 3, 4, 5, 6, 7, 8

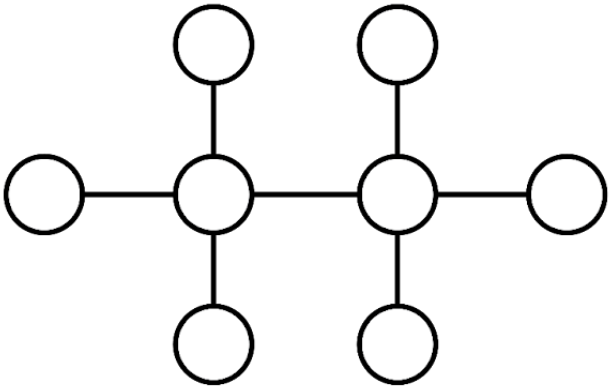


Graceful Graphs Challenge #3: Caterpillars

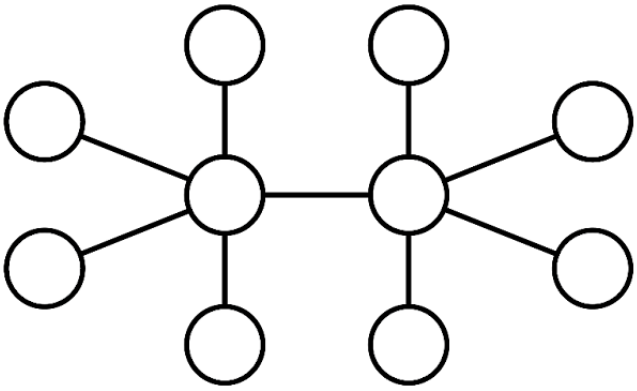
Number List:
0, 1, 2, 3, 4, 5



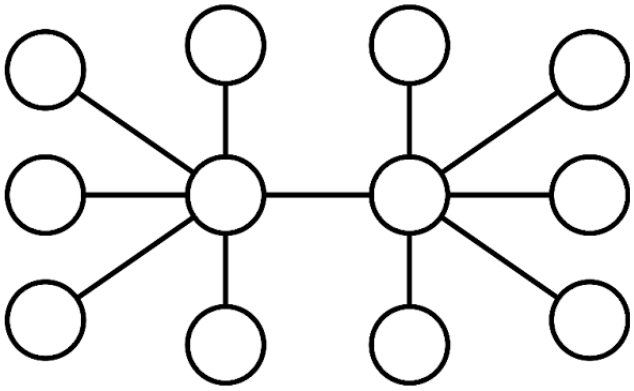
Number List:
0, 1, 2, 3, 4, 5, 6, 7



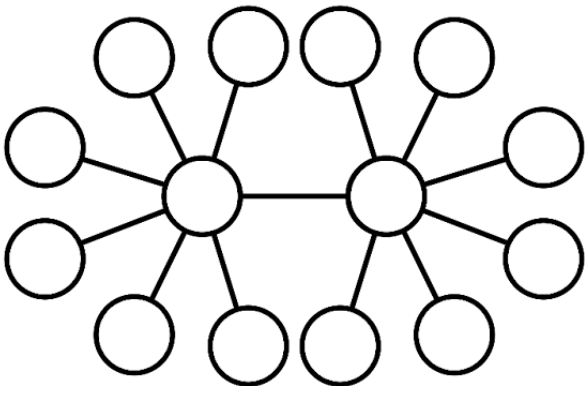
Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9



Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11



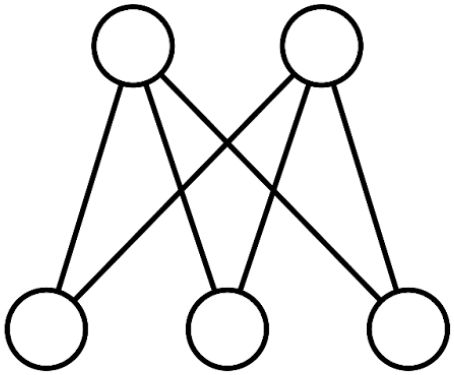
Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13



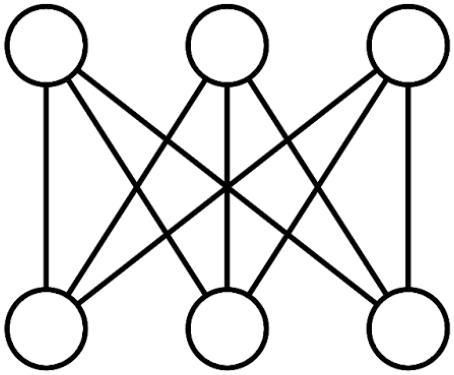
Graceful Graphs Challenge #4: Spotlights

For these puzzles you won't use every number in the number list.

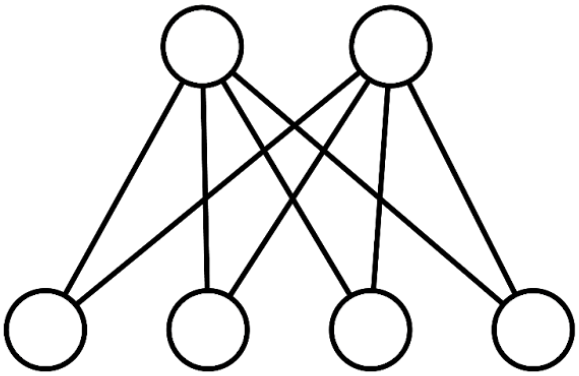
Number List:
0, 1, 2, 3, 4, 5, 6



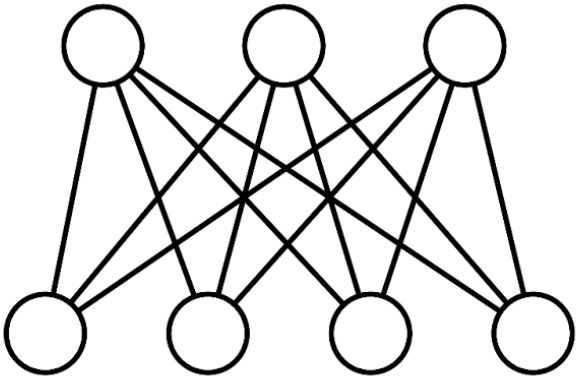
Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9



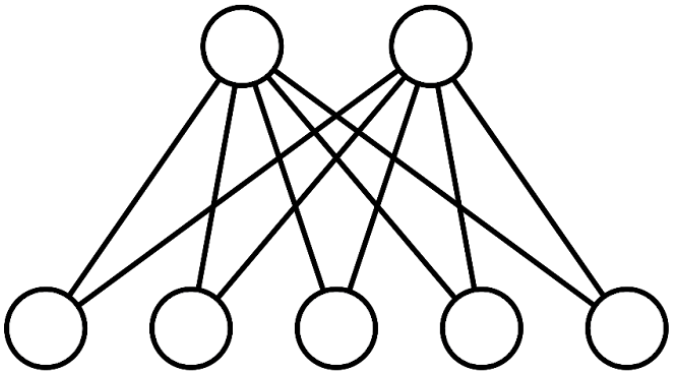
Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8



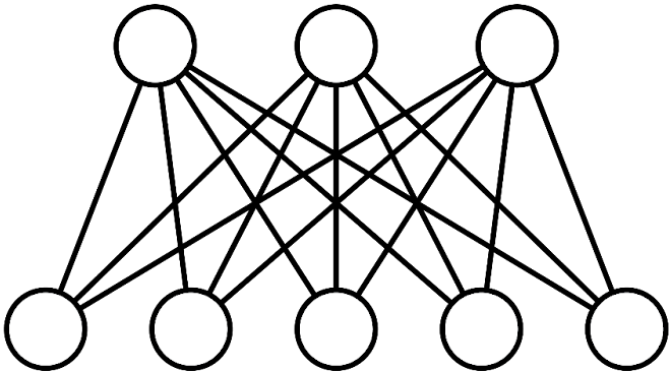
Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12



Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10



Number List:
0, 1, 2, 3, 4, 5, 6, 7, 8,
9, 10, 11, 12, 13, 14, 15



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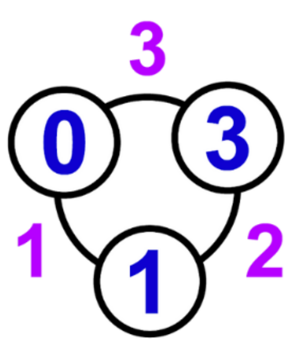

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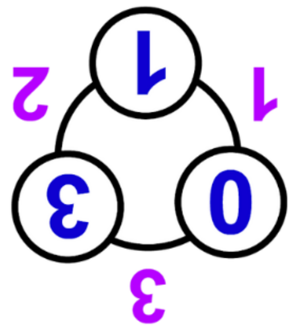

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