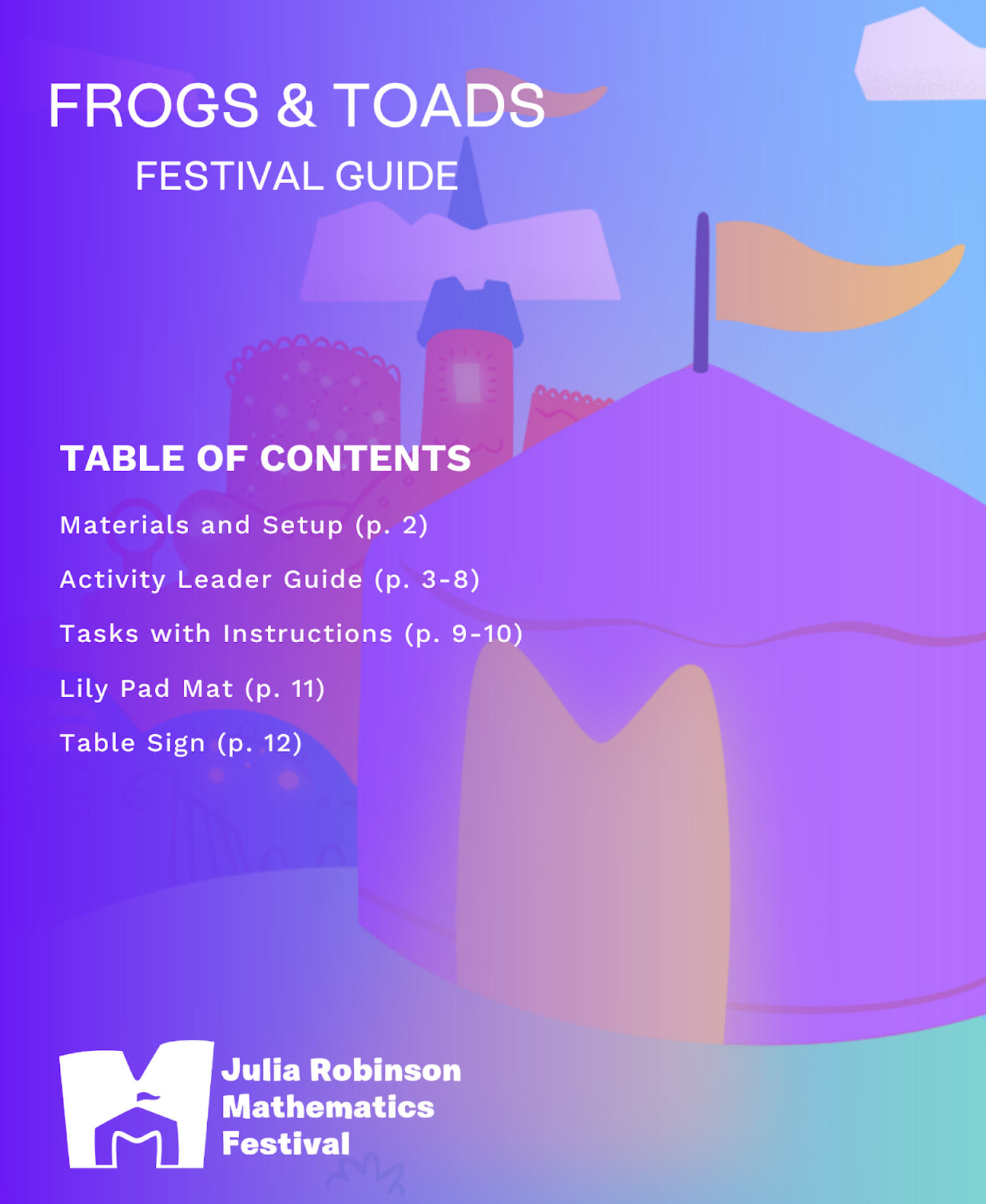
FROGS & TOADS

FESTIVAL GUIDE



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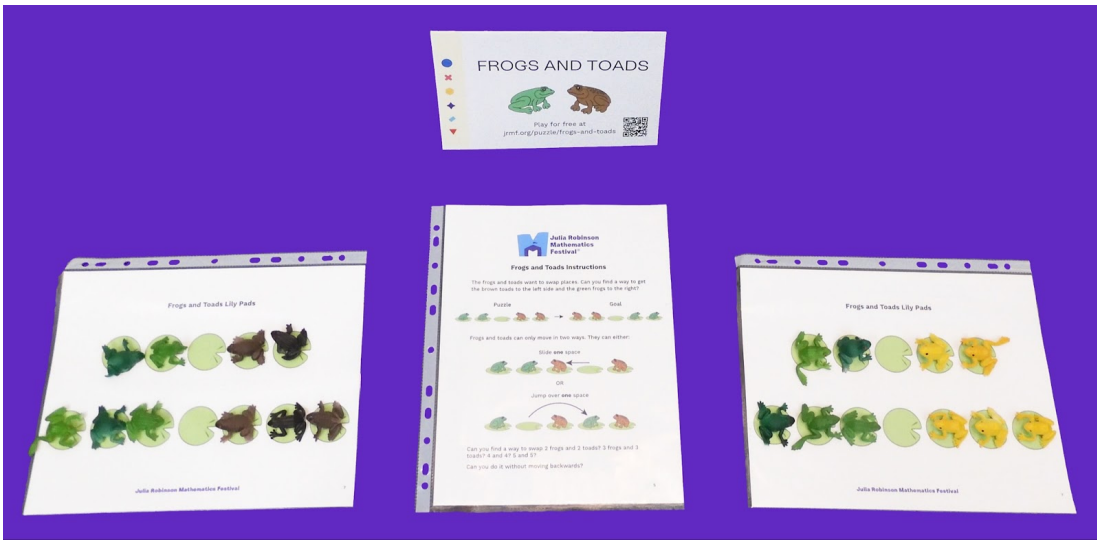
Materials and Setup

Per table (assuming 5 students per table), you will need:

Per Table	Material Preparation	
50 frogs	Prepare 5 sets with 5 frogs of one color and 5 'toads' of a second color.	
5 copies of Tasks with Instructions	2-page sheet *can be printed double-sided*	p. 9-10
5 copies of Lily Pad Mat	1-page sheet	p. 11
1 copy of Table Sign	1-page sheet *print on cardstock for sturdiness*	p. 12
Paper and pencils		

Per Table	Purchasing Materials		
50 frogs	pack of 60 for $13.99		Frogs are fun to play with. However you can also use other distinguishable items like two-color counters, coins, etc.
10 plastic sheet protectors	pack of 100 for $7.67	pack of 500 for $26.99	These are recommended in order to protect the documents that students will be handling.



Objective

The goal is to move all of the toads on the lily pads to the left and all of the frogs to the right.

Rules:
1. Each turn one frog or one toad can:
    a. Slide one space.
    b. Jump over one space.

Materials

Each Frogs and Toads table should be prepped for 5 stations.
Each station needs:
1. 4 frogs of one color and 4 toads of a second color.
2. Frogs and Toads tasks with instructions.
3. Frogs and Toads lily pad mat.
4. Paper and pencils for tracking moves.

How to Play

Introduce the activity without overexplaining it and without telling what strategies students might want to use. As much as possible, avoid giving away answers. Students should be encouraged to explore, experiment, and learn from their mistakes.

1. Demonstrate the rules by playing a game with the student.
2. Encourage them to explain their thinking out loud as they choose which move to make.
3. Have the student explore the game, starting with the 2 frogs, 2 toads, and 5 lily pads.

Standards

1. Make sense of problems and persevere in solving them. CCSS.MP1
2. Construct viable arguments and critique the reasoning of others. CCSS.MP3
3. Look for and make use of structure. CCSS.MP7
4. Look for and express regularity in repeated reasoning. CCSS.MP8

Asking Good Questions

1. Ask questions about confidence.
    a. When a student asks you “Is this right?”, instead of saying “yes” or “no” right away, ask them how confident they are in their answer. Here are some examples:
        i. “Maybe. What do you think? How confident are you?”
        ii. “On a scale of 1-5, how confident are you in your answer?”
    b. If a student is not confident in their answer, follow up by asking “What would help you feel more confident in your answer?” or “Why do you not feel confident?” This helps you determine how best to help the student through their explorations.

2. Ask students about choices.
    a. When a student is stuck or shows you a wrong answer, instead of jumping in and showing the student the correct answer, start by asking about the choices that the student made along the way. Here are some suggested steps to follow:
        i. Start from the beginning.
        ii. Ask students to show you what they’ve tried so far.
        iii. When the student gets to a point where they have different choices, ask the student “What other choices can you make here?”
        iv. Have the student make a different choice and try to solve the puzzle. This helps the student see that they have the power to make different choices during an activity, and they’ll start to do this on their own in the future.
        v. If you’re familiar with the puzzle or a particular solution, stop the student only when a different choice will help them get to the solution. This will help them feel successful faster without you giving away too much of the answer.

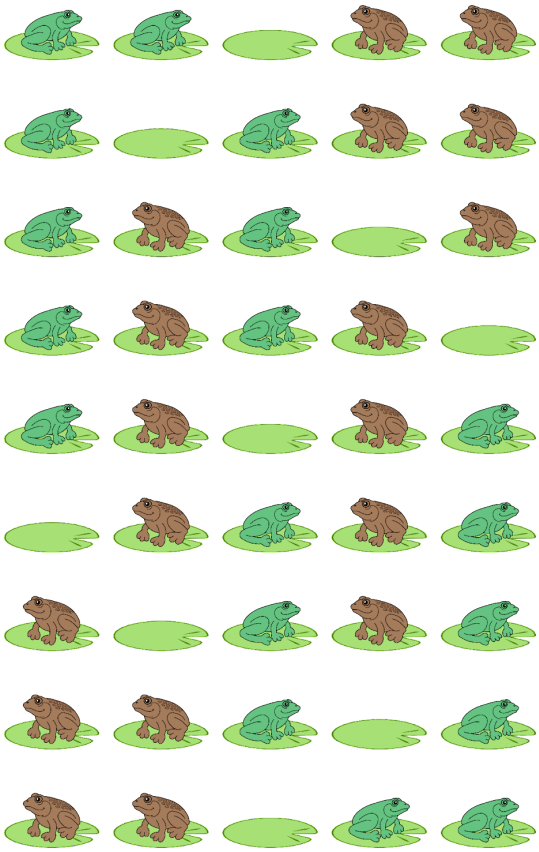
3. Ask students about strategies.
    a. If a student is getting into the activity and has been doing it for a while, ask the student if there are any strategies they’ve come up with to help them solve the puzzle or win the game.
    b. Follow up by asking if they think their strategies will work for all puzzles and/or larger puzzles, more complex puzzles, etc. Have the student explore more complex puzzles to test out their strategies.
    c. This is a great way to encourage a student to dive deeper into an activity and to start looking for patterns, structure, and proofs.

4. Activity specific questions.
    a. How many types of moves are there? How can you distinguish between them?
    b. Does it matter which end you start at?

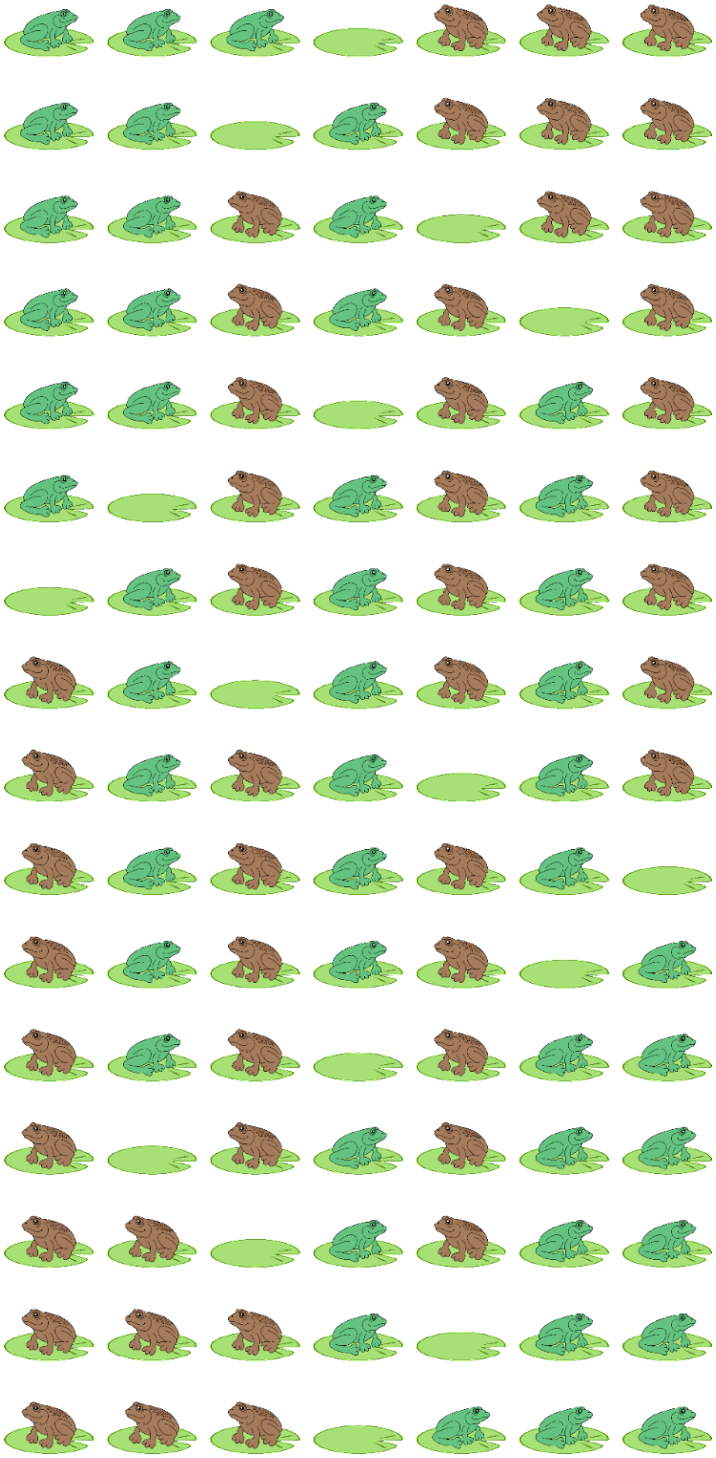
Answers

Most students will be able to solve these puzzles through trial and error if they allow themselves to move backwards. The challenge of solving these puzzles without moving backwards is much harder. Below are example solutions when there are 2 frogs and toads and 3 frogs and toads.

2 Frogs and Toads Answer



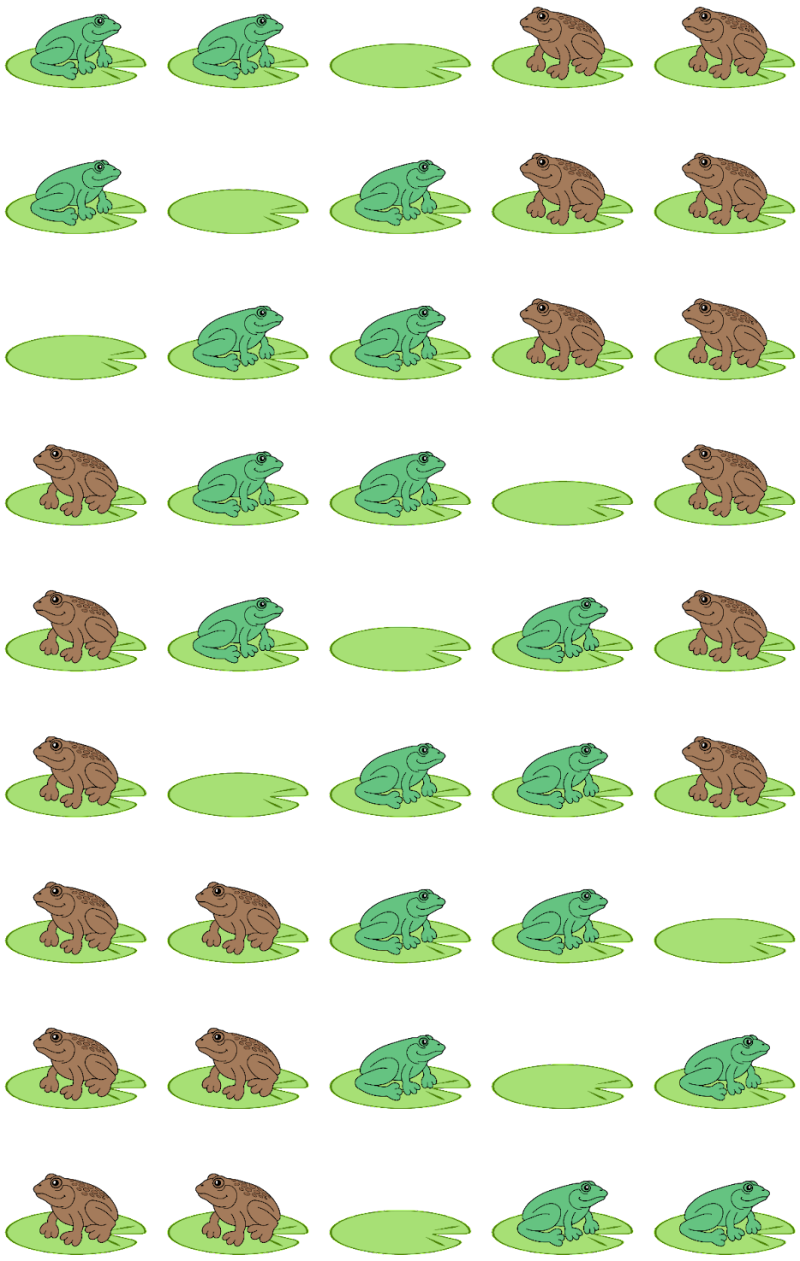
3 Frogs and Toads Answer



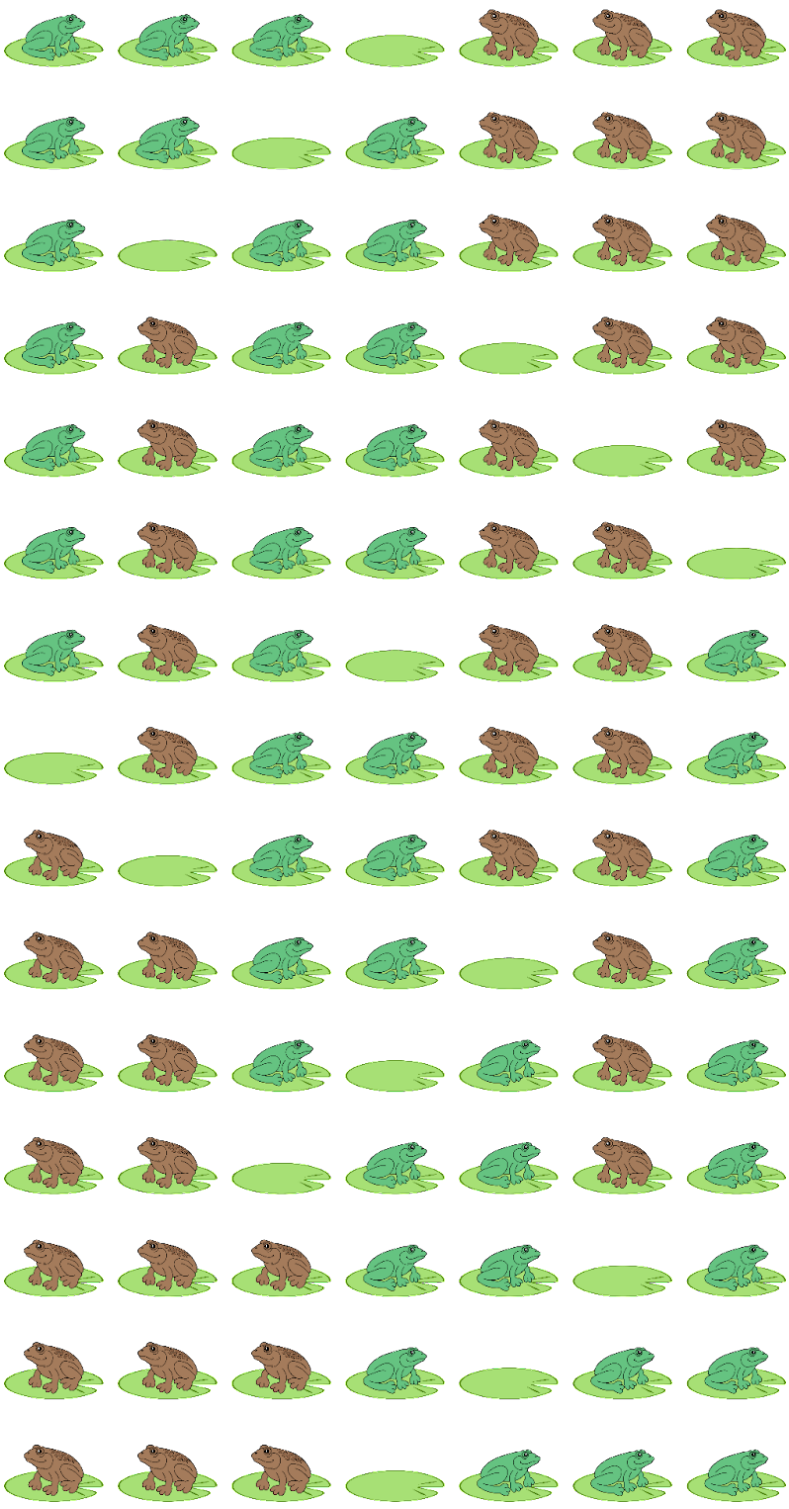
The solutions for 4 frogs and toads and 5 frogs and toads are similar to the solutions above. The strategy for all of these solutions involve weaving the frogs and toads together, so that you reach a point where you have frog, toad, frog, toad, frog, toad, etc.

The solutions for the Frogs and Toads Challenge puzzles – **in which frogs and toads now must jump two spaces instead of one** – with 2 frogs and toads and 3 frogs and toads are shown below.

2 Frogs and Toads Challenge Answer



3 Frogs and Toads Challenge Answer



The solutions for the Frogs and Toads Challenge puzzles don't follow as clear of a pattern as the solutions for the main activity. The solutions for the challenge with 4 frogs and toads and 5 frogs and toads are left as something for you to explore along with your students.

An Extra Challenge (this section is not necessary to successfully lead this activity)

As an extra challenge, ask students to predict how many moves they think they will need to solve each puzzle. For the main task (in which the frogs and toads can only jump one space), students may notice a pattern based on square numbers:

- 2 frogs and 2 toads - 8 moves
- 3 frogs and 3 toads - 15 moves
- 4 frogs and 4 toads - 24 moves
- 5 frogs and 5 toads - 35 moves

For each puzzle, the number of moves that a student needs is one less than a square number. This pattern does continue, and there is a quadratic equation that predicts the number of moves based on the number of frogs and toads:

If there are n frogs and n toads, then $(n+1)^2 - 1$ is the total number of moves.

Some teachers use Frogs and Toads (also known by other names) to introduce quadratic equations.

A nice property of this quadratic equation is that it is not too hard to derive from the rules of the puzzle. Although not appropriate for most students, you can have students who are familiar and experienced with problem solving and proofs try to derive this equation on their own. Below is one way to do this:

$Proof$: Frogs and toads can either jump or slide, so the total number of moves is the sum of the number of jumps and number of slides.

$$total\ \#\ of\ moves\ =\ \#\ of\ jumps\ +\ \#\ of\ slides$$

Every frog must switch places with every toad at some point, either by having the frog jump over the toad or having the toad jump over the frog. Because each of the n frogs must switch places with each of the n toads, there must be n^2 jumps.

To find the number of slides, we're going first look at the total number of spaces the frogs and toads need to move. Each frog and toad needs to move $n+1$ spaces. There are $2n$ frogs and toads, so they need to move a total of $2n(n+1)$ spaces.

When a frog or toad jumps, it moves two spaces. When a frog or toad slides, it moves one space. Combining all of these ideas together gives us the following equation:

$$total\ \#\ of\ spaces\ =\ 2\ (\#\ of\ jumps)\ +\ 1\ (\#\ of\ slides)$$

$$2n\ (n+1)\ =\ 2\ (n^2)\ +\ \#\ of\ slides$$

By solving for the number of slides, we find that $\#\ of\ slides\ =\ 2n$. Plugging this into the original equation gives the total number of moves:

$$total\ \#\ of\ moves\ =\ \#\ of\ jumps\ +\ \#\ of\ slides\ =\ n^2 + 2n\ = n^2 + 2n + 1 - 1 =\ (n+1)^2 - 1$$

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Frogs and Toads Instructions and Tasks

The frogs and toads want to swap places. Can you find a way to get the brown toads to the left side and the green frogs to the right?

Puzzle

Goal

Frogs and toads can only move in two ways. They can either:

Slide **one** space

OR

Jump over **one** space

Can you find a way to swap 2 frogs and 2 toads? 3 frogs and 3 toads? 4 and 4? 5 and 5?

Can you do it without moving backwards?

Frogs and Toads Challenge

The goal is the same: the frogs and toads want to swap places.

Puzzle

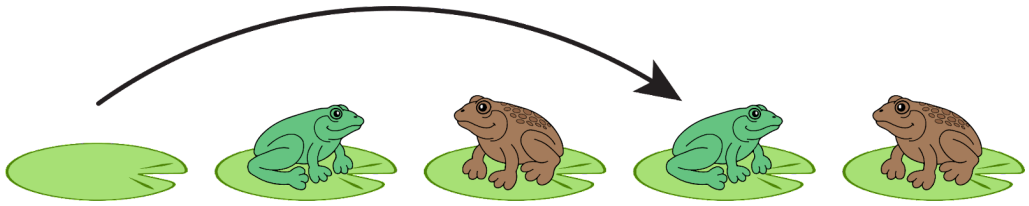
Goal

But now, **frogs and toads cannot jump over only one space.** Instead, frogs and toads can either:

Slide **one** space

OR

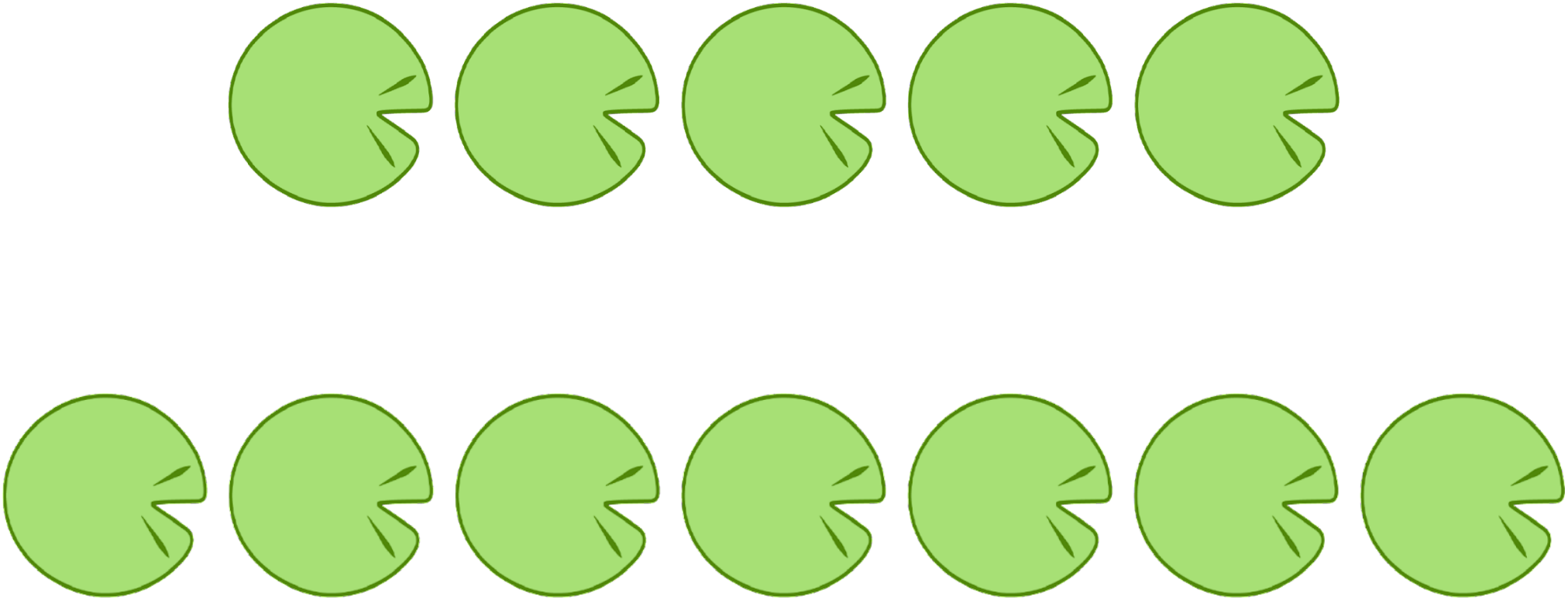
Jump over **two** spaces



Can you find a way to swap 2 frogs and 2 toads? 3 frogs and 3 toads? 4 and 4? 5 and 5?

Can you do it without moving backwards?

Frogs and Toads Lily Pads



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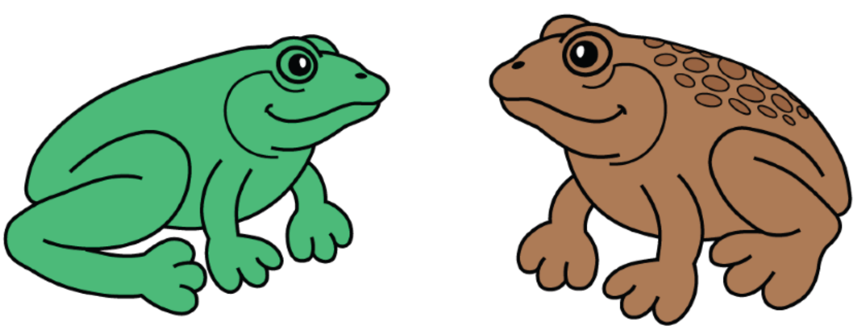

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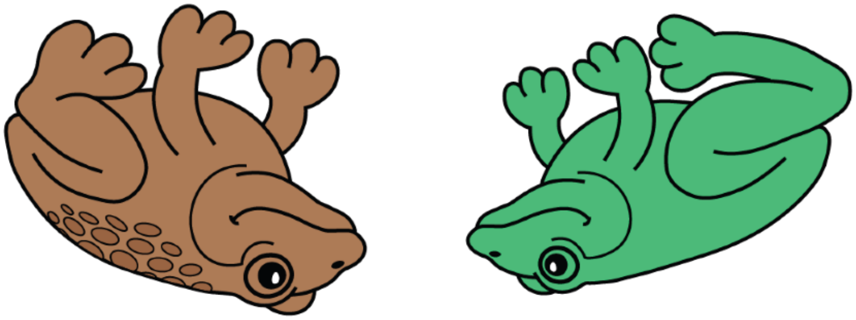

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