Objective

Trace each doodle without lifting your marker.

Rules:
1. Start at any point.
2. Trace the whole doodle without lifting your marker.
3. You cannot trace a line more than once.

Introduction

Have the first two images ready to show the students.

Explain
Have the students offer suggestions for how to trace the first image. Pretend to misunderstand and make some purposeful mistakes (such as going over the same line twice) in order to reinforce the rules.

Engage
Ask a student to try tracing the second image. Encourage them to explain their thinking out loud as they trace.
If their trace leads to a position where it's impossible to complete, have them explain why it's impossible. Encourage them to begin again.

Common misconceptions
Students might think that:
   a. They have to start at the top right corner.
   b. They aren't allowed to go through the same vertex more than once.

Exploration

In pairs, have your students explore the rest of the puzzles by tracing a path on the plastic sheet protector using a dry erase marker.

Circulate and ask questions to encourage deeper thinking:
1. Which tasks did you find the hardest so far? The easiest? Why?
2. Have you found a strategy that works for all or many of the tasks?
3. When a child is stuck, ask:
   a. Can you try a different starting point?
   b. What do you know so far?

  c. What didn't work?
  d. What are you thinking about trying?
4. To support pattern recognition, ask:
  a. For each puzzle, circle your start and end points. What do you notice?
  b. Which puzzles were impossible? What do you think made them impossible?
  c. Can you predict whether a puzzle is possible before solving it? How do you know?
  d. In this puzzle [*choose an impossible puzzle*], what could you change to make it possible?
5. "Tell me more." is a great basic prompt for getting a child to explain their thinking.

Extend
1. Have student create their own tracing puzzles for others to try.

Discussion

As a group, have students share something about their experience with Doodles. Try to have at least 3 students share out. Variations of the questions asked earlier are great for generating discussion, such as:
  a. Do you have a strategy that worked for more than one puzzle?
  b. How do you know if a puzzle is impossible?

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Materials

1. Doodles tasks sheets pp. 6-11
2. Dry erase sleeves
3. Dry erase markers

Optional: Doodles instructions sheet p. 5

Assessment

Evidence of student learning during problem-solving activities can be obtained from three sources: observations, conversations, and products.

Observation involves actually observing students while they perform tasks and demonstrate skills and may take the form of a checklist or quick note.

Conversation involves engaging students in discussion that encourages them to articulate what they are thinking and then capturing that with a quick note.

Products are student-created records that capture not only their answer, but some of the process that led them to the answer.

Standards

1. Make sense of problems and persevere in solving them.
   CCSS.MP1
2. Construct viable arguments and critique the reasoning of others.
   CCSS.MP3
3. Model with mathematics.
   CCSS.MP4
4. Look for and make use of structure.
   CCSS.MP7

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General Answers:

Note: This is for your information only. Although some students might notice that whether a puzzle is solvable is somehow related to the vertices, they are not expected to know/learn the language.

In school, the term "graph" usually refers to the *x* and *y* coordinates on a Cartesian plane. In graph theory, a graph is any set of points with lines connecting some of them. A point is called a vertex and a line connecting two vertices is called an edge. The number of edges connected to a vertex is called the degree of that vertex.
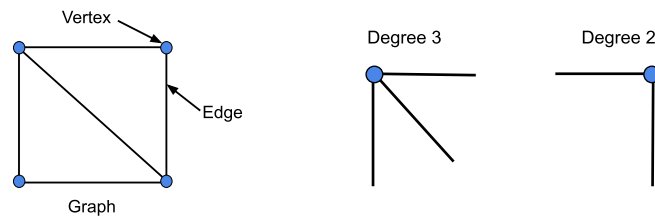
1. If an image has no vertices with an odd degree (i.e. a point with an odd number of lines connected to it), it is possible to trace starting at any vertex.
2. If an image has two vertices with an odd degree, it is only possible to trace starting at one of these two vertices.
3. If an image has any other number of vertices with an odd degree, it is impossible to trace.

Answers to Challenges 1 to 6:

1. All doodles in Challenge 1 can be traced. For each doodle, there are exactly two possible starting points.
2. All doodles in Challenge 2 can be traced. For each doodle, there are exactly two possible starting points.
3. The first three doodles in Challenge 3 can be traced. For each of these doodles, there are exactly two possible starting points. The bottom right doodle is impossible to trace.
4. The first three doodles in Challenge 4 can be traced. For the top left doodle, it is possible to trace the doodle starting at any point. For the other two possible doodles, there are exactly two possible starting points. The bottom right doodle is impossible to trace.
5. All doodles in Challenge 5 can be traced. For the top left and bottom doodle, it is possible to trace the doodle starting at any point. For the other doodles on the page, there are exactly two possible starting points.
6. All doodles in Challenge 6 can be traced. For the bottom right doodle, it is possible to trace the doodle starting at any point. For the other doodles on the page, there are exactly two possible starting points.

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Doodles Instructions

Trace each doodle without lifting your marker.

Doodles Rules:

1. Pick any point to start on.

2. Trace the whole doodle without lifting your marker.
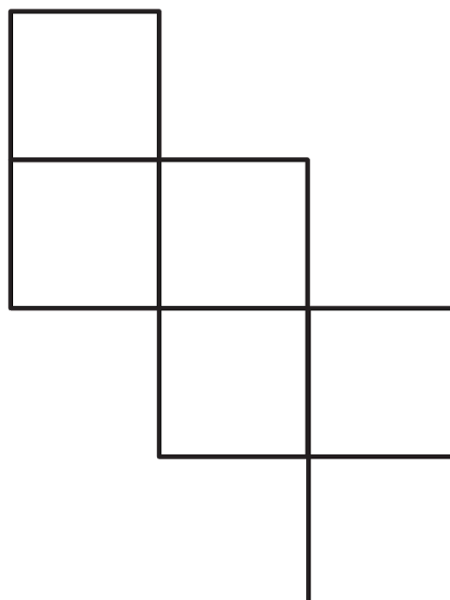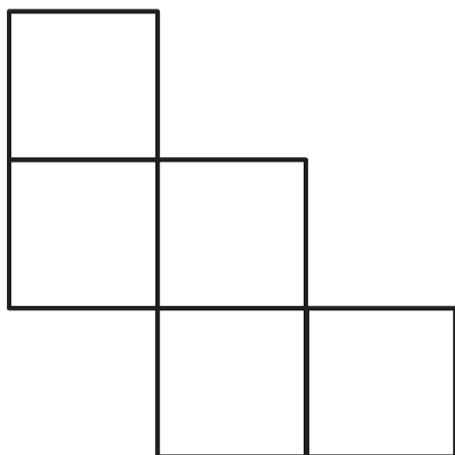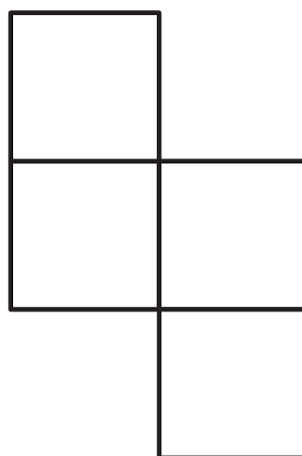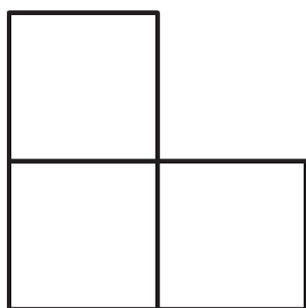
3. You cannot trace a line more than once.
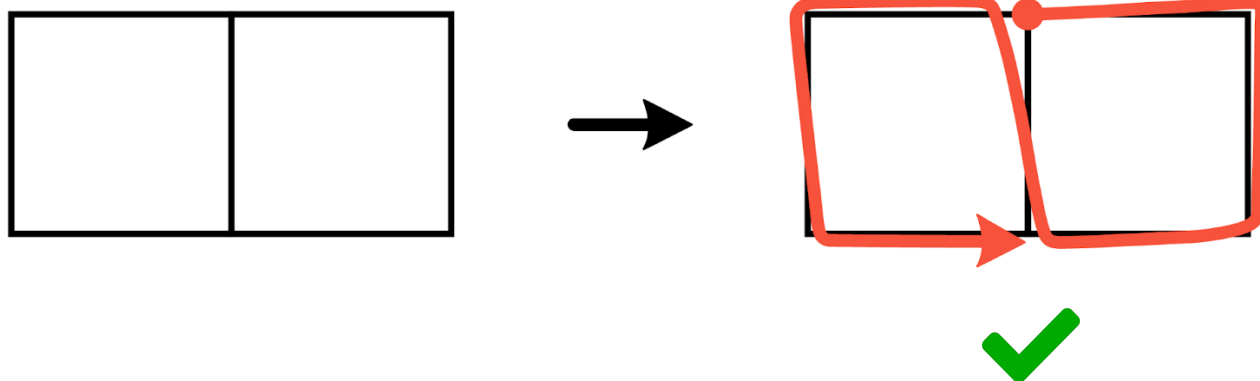
Doodles Tasks

Challenge 1
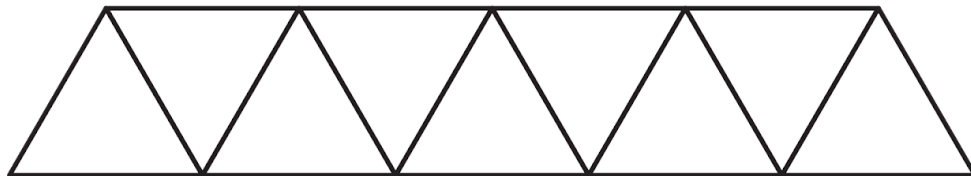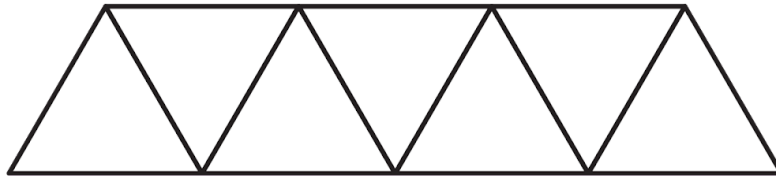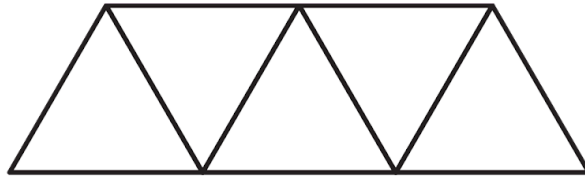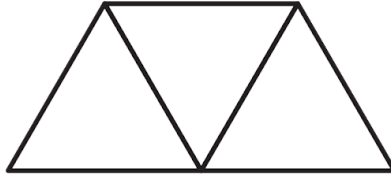
Can you trace each doodle without lifting your marker?

Can you find more than one way to trace each doodle?

Challenge 2
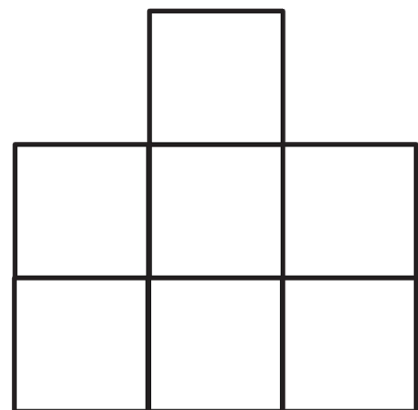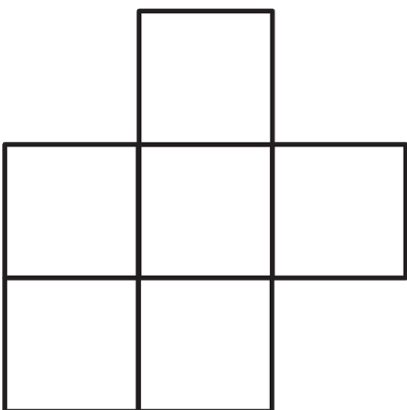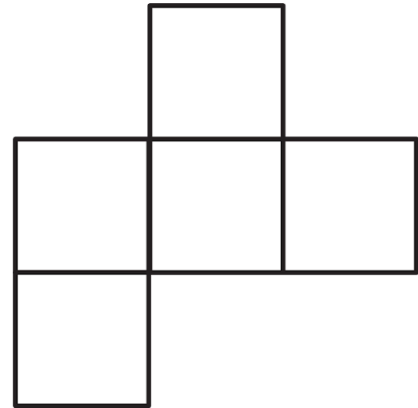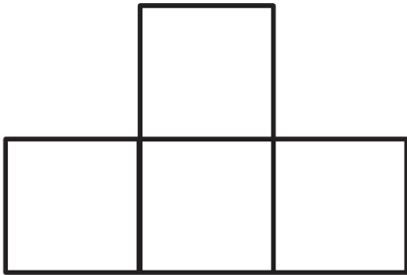
Can you trace each doodle without lifting your marker?

Circle the points that you start and end on. Do you notice a pattern?

Challenge 3

One of these doodles is impossible to trace without lifting your marker. Which one is impossible?

Circle the points that you start and end on. What makes these points special?
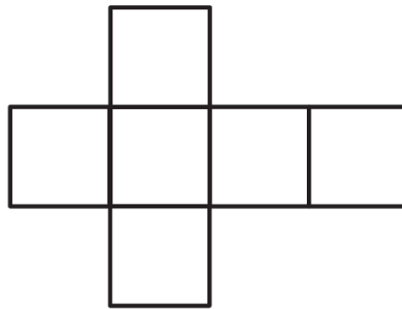
Challenge 4

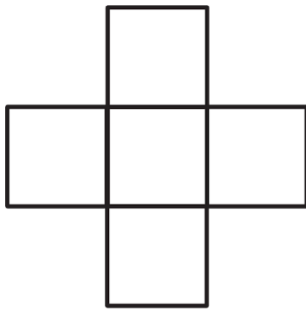Is it possible to trace each of these doodles?

For the doodles you can trace, can you start on any point?
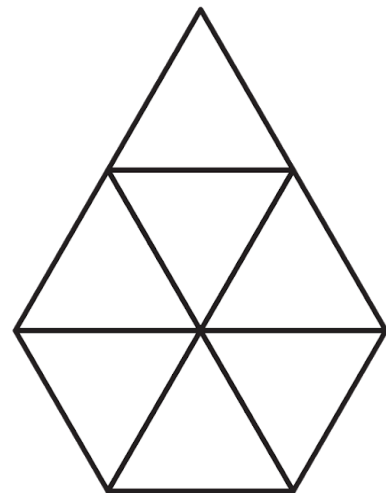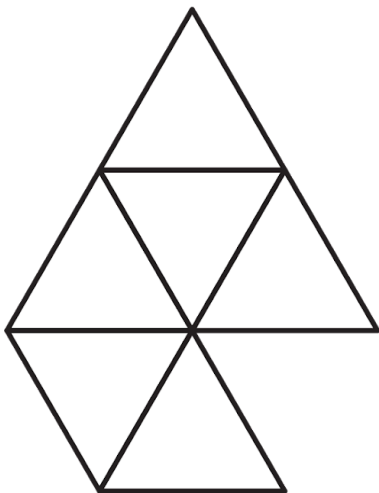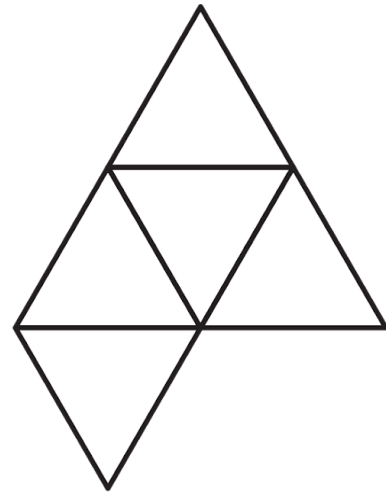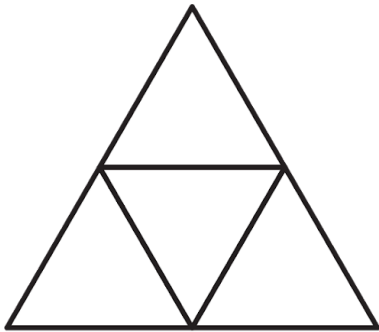
Challenge 5

Is it possible to trace each of these doodles?

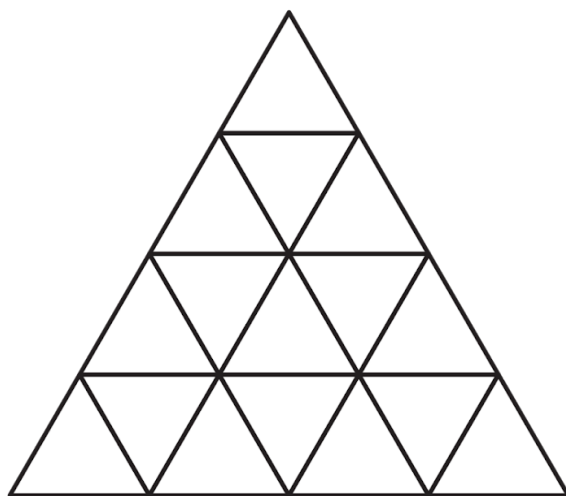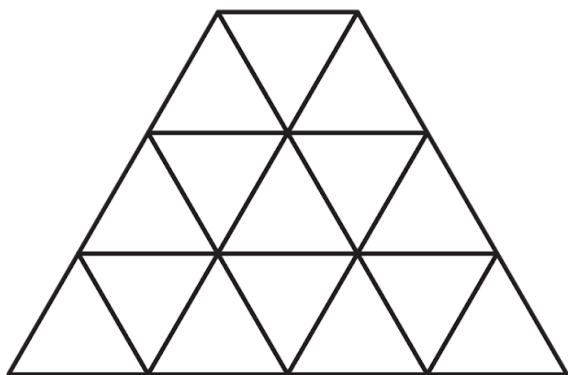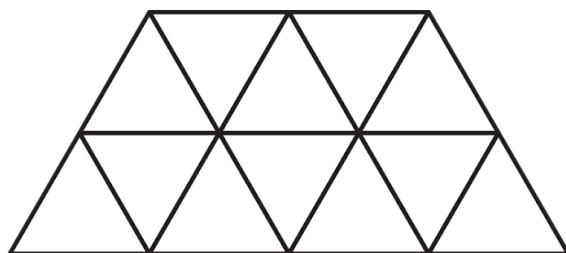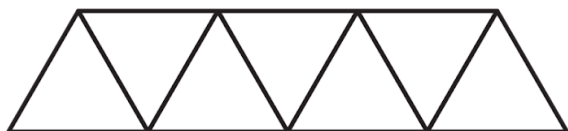For the doodles you can trace, can you start on any point?
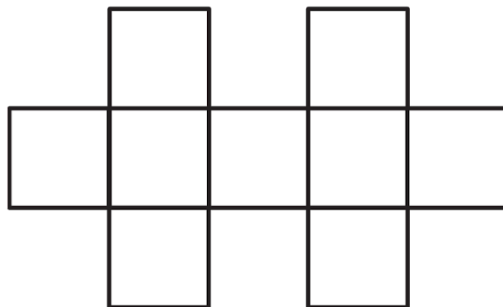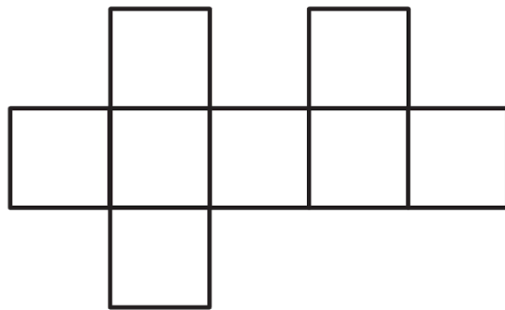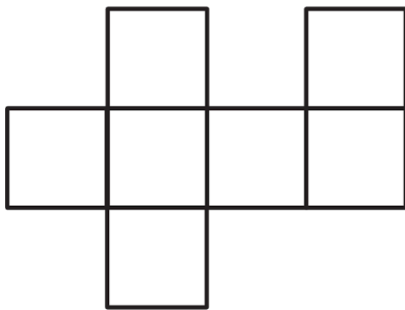
Question 6

Is it possible to trace each of these doodles?

Can you find a pattern that helps you trace the last doodle quickly? How would you explain that pattern to a friend?

Is it possible to trace triangles that are larger than the last doodle? How would you do it?