Objective

Begin with a row of cups and end with all of the cups in a single stack.

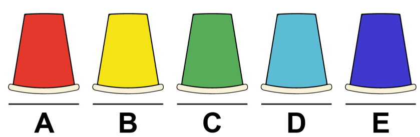
Rules:
1. Count the number of cups in a stack. That stack must jump that number of spaces. For example, 1 cup can only move 1 space; 2 cups have to move 2 spaces; 3 cups have to move 3 spaces...
2. A cup or stack of cups cannot move into an empty space. They have to land on another cup or stack of cups.

Introduction

Have ready to go a set of 5 cups laid out on the row sheet.

Explain

Tell the students: “We want to stack all these cups all on top of each other. Pretty easy, right? You just pick one up and put it on top of another one and do that over and over. But that’s not quite how this activity works!”



Using only the first 5 spaces on the row sheet, demonstrate the rules by moving some of the cups.

Engage

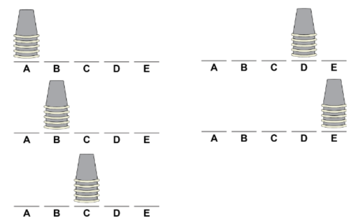
1. Ask the class for suggestions to finish stacking all the cups in a single stack according to the rules. Encourage them to explain their thinking for the moves they suggest.
2. If their suggestions lead to a position where it's impossible to stack the rest of the cups, have them explain why it's impossible. Either start over, or take back the last move, and see if they can suggest a different move.
3. (If you’re printing the tasks sheets, explain how to use them).

Common misconceptions

Students might think that:
   a. They can land on empty spaces.
   b. They can pick up cups they jump over.
   c. They can only move one stack, e.g., they will think that if they start with the red cup, then they can only move the stack with the red cup in it.

Exploration

In pairs, have your students explore, starting with a row of 5 cups. When the pair thinks they have a strategy, have them demonstrate their solution. Next have them explore:



1. Can they find a way to stack the cups but with the cups ending in a different space each time?
2. Can they stack longer rows of cups and end in each space?

Circulate and ask questions to encourage deeper thinking:

a. Is it possible to land on all the spaces?
b. Which space was easiest to land on? Hardest?
c. Does your strategy for stacking cups on one of the spaces help you stack the cups on any of the other spaces? (The goal here is for students to see symmetry in their moves).
d. Is there a strategy for stacking all of the cups that you think will work if we add one more cup? Two cups? Any number of additional cups?
e. If we add one cup, do you think it's possible to end up on any of the spaces? Two cups? No matter how many cups we have, do you think it's possible to end up on any of the spaces?
f. When a student is stuck, ask:
    i. What do you know so far?
    ii. What didn't work?
    iii. What are you thinking about trying?
g. To support pattern recognition, ask:
    i. What if you track your moves?
    ii. How many moves did it take? Can you do it in fewer?
    iii. Can we ask a simpler question?
    iv. What do you think will happen?
    v. Is there a way to organize what we know to understand it better?
h. "Tell me more." is a great basic prompt for getting a student to explain their thinking.

Extend

1. Do their solutions for longer rows generalize? Can they now describe how to end in each position with an *n*-cup puzzle?
    a. If they can end in position *p*, can they say something about what the bottom cup in the final stack has to be?
2. Return to playing with 5 cups, but this time focus on what *order* the cups are stacked in. (You may want to have them label the cups 1, 2, 3, 4, and 5.)
    a. For example, point out: You were able to stack the cups so that [describe the color order that the student stacked their cups]. What other different stacks can you make? Can you order the cups in any way you want? [Give the student some challenges, like red, yellow, green, light blue, dark blue, etc.]
    b. For a given ending position, are there any other things they can say about the ordering of the end stack? (Things like "the top cup must be cup 3" or "cup 2 must

be above cup 4," for example.)

c. For which numbers of cups is it possible to win with a stack in counting order (1, 2, 3, ..., *n-1*, *n* or *n*, *n-1*, ..., 3, 2, 1)?

3. Place 6 cups in a 2x3 grid. A stack of *k* cups can still only move exactly *k* spaces, but it can do so in any combination of vertical and horizontal movements as long as they only move left or right (not both) and they only move up or down (not both).
   a. Which positions can they win in? Are there any that are impossible?
   b. There is symmetry in a 2x3 board. Can they use that to more quickly decide which positions they can end in?
   c. What can they say about a 2x4 board? 2x5? Are there any conclusions they can draw for a 2x*n* array of cups?
   d. In general, what can they say about an *m*x*n* array of cups? Is it possible to win in every position or not? If so, how? If not, why not?

Discussion

As a whole group, have students share something about their experience with Cup Stacking. Try to have at least 3 students share out. Variations of the questions asked earlier are great for generating discussion, such as:

a. What strategies did you come up with? Did they always work?
b. What did you try when you first started? What didn't work?
c. What did they notice about the order of the cups?
d. For those who explored the 2-D grid, describe the strategy they tried. Did what they learned from the 1-D rows help at all?

Materials

1. Colored, stackable cups.
2. Row sheets (pp. 11-13) taped together for placing the cups.

Optional: Cup Stacking instructions sheet p. 7
Cup Stacking tasks sheet pp. 8-9
Cup Stacking extensions instructions and tasks sheet p. 10
Cup Stacking extensions 2-D grids pp. 14-15

To explore the activity yourself, you can try our digital version here: jrmf.org/activities/cup-stacking

Assessment

Evidence of student learning during problem-solving activities can be obtained from three sources: observations, conversations, and products.

Observation involves actually observing students while they perform tasks and demonstrate skills and may take the form of a checklist or quick note.

Conversation involves engaging students in discussion that encourages them to articulate what they are thinking and then capturing that with a quick note.

Products are student-created records that capture not only their answer, but some of the process that led them to the answer.

Standards

1. Make sense of problems and persevere in solving them.
   CCSS.MP1
2. Construct viable arguments and critique the reasoning of others.
   CCSS.MP3
3. Model with mathematics.
   CCSS.MP4
4. Look for and make use of structure.
   CCSS.MP7

Answers

General Answers:

1. For any row of cups, it is possible to end in any position.
   a. Choose a target ending position.
   b. Treat the cups to the left of the target as a subpuzzle and end with them in position 1.
   c. Treat the cups to the right of the target as another subpuzzle and end with them in the last position.
   d. Jump the stacks in position 1 and the last position onto the target position.

2. For any row of cups, the fewest moves is *n-1* (one less than the number of cups).

3. The only rows of cups that can be ordered from 1 2, 3...*n*-1, are 1, 2, 4, 10, 32, 78,...

Possible Student Answers to Task sheet questions 1 to 6:

Note: These are only possible answers, actual student answers will vary.

1. Can you find a way to stack the cups on each of the five positions?
   - For any row of cups, it is possible to end in any position.

2. One of these stacks is impossible to make. Can you figure out which one? How would you explain to a friend why it's impossible to make?
   - C is impossible because the bottom cup has to have the same number as its position.

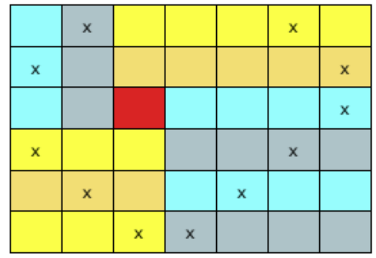
3. One of these stacks is impossible to make. Can you figure out which one?
   - B is impossible because you would not be able to make a jump of the size you need to accomplish it.

4. What is another stack that you think is impossible to make? Explain why you think it's impossible to make.
   - e.g., 54321 because it's a mirror of the stack in question 4, which we already know is impossible.

5. Now try six cups. Can you stack the cups on each of the six different positions? Can you find a stack that's impossible to make?
   a. For any row of cups, it is possible to end in any position.
   b. various answers

6. Now you choose how many cups you want to use. Can you stack the cups on each of the different positions? Can you find a stack that is impossible to make?
   - various answers

General Answers to Extension questions:



1. For a 2D grid, there is an approach based on 1D solutions and their flexibility in allowing all ending positions:
   a. Choose a target ending position (*red square*).
   b. Divide the board into four (or fewer, if the target is on an edge or corner) rectangles swirling around the target position and stripe each rectangle in the direction of its longest dimension (*s*) as pictured here.
   c. Within each stripe, count the number of spaces (*s)* and find the space in the stripe that is distance (*s)* away from the target square, as marked with x's in the picture below.
   d. Use your 1D strategy to stack all cups within each stripe on its Xed position.
   e. Jump your stacks from the exed spaces to the target space.

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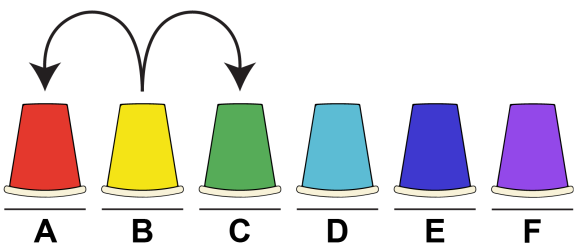
This activity was made in collaboration with sfmathcircle.org and mathpickle.com

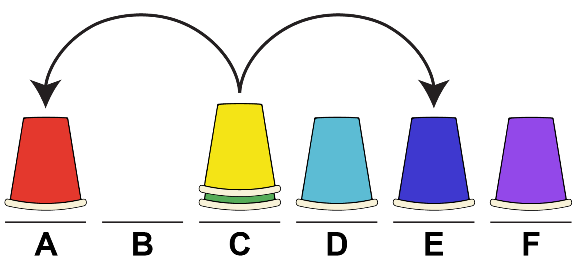
Cup Stacking Instructions

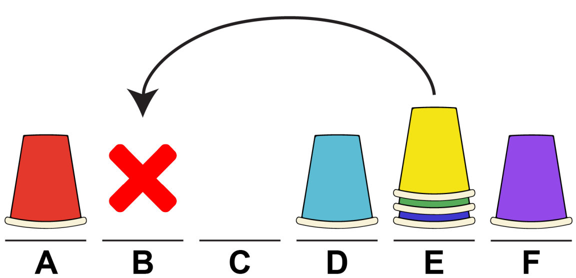
Can you stack all of the cups into a single stack?

Rules:

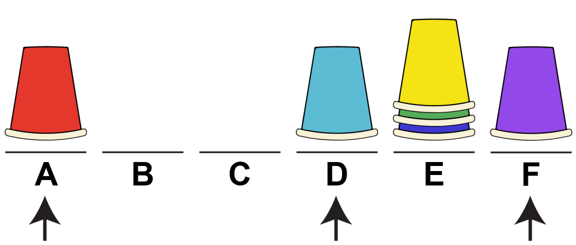


1. Count the number of cups in a stack. That stack must jump that number of spaces.





2. Cups cannot land on an empty space.

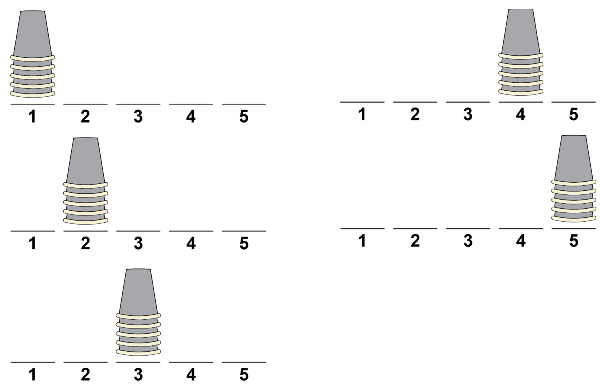


3. You can move any stack you want. How would you finish stacking these cups?

Cup Stacking Tasks

Challenge 1

Can you find a way to stack the cups on each of the five positions?

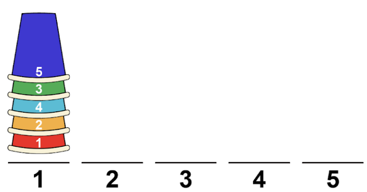


Challenge 2

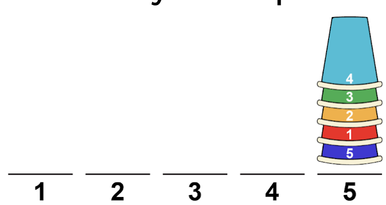
One of these stacks is impossible to make. Can you figure out which one?

How would you explain to a friend why it's impossible to make?

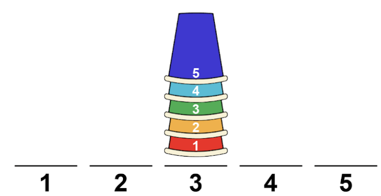
A)



B)

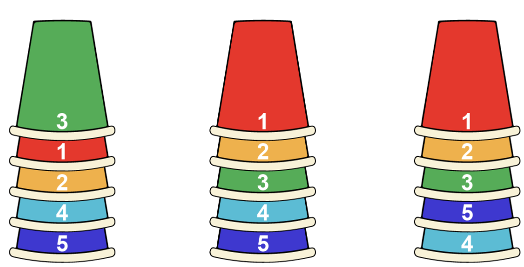


C)



Challenge 3

One of these stacks is impossible to make. Can you figure out which one?

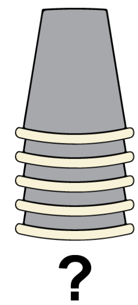


A **B** **C**

Challenge 4

What is another stack that you think is impossible to make?

Explain why you think it's impossible to make.

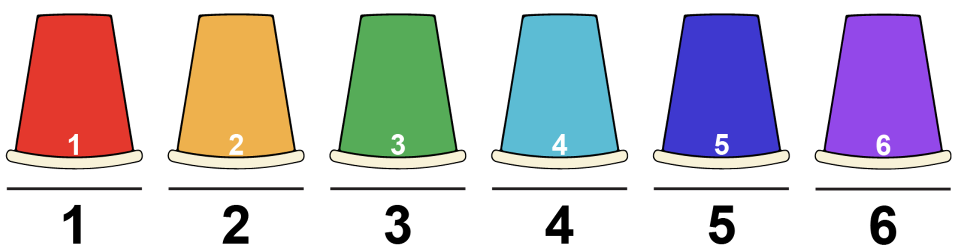


Challenge 5

Now try six cups.

Can you stack the cups on each of the six different positions?

Can you find a stack that's impossible to make?

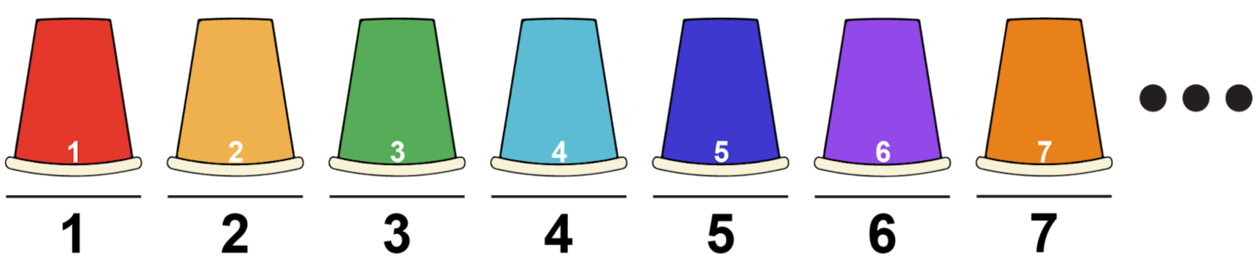


Challenge 6

Now you choose how many cups you want to use.

Can you stack the cups on each of the different positions?

Can you find a stack that is impossible to make?

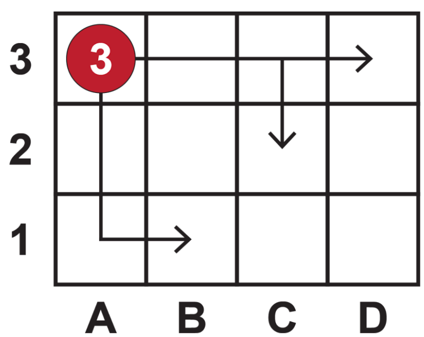


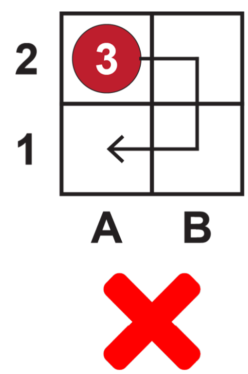
Cup Stacking 2-D Challenge

In Cup Stacking 2D, cups can move left or right and up or down the number of squares equal to the number of cups on that square.

Backtracking isn't allowed!

See below for some examples of possible moves for a stack of 3 cups.





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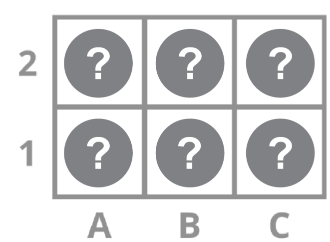
1. Start with a 2 x 3 grid.

    a. Can you find a way to stack all of the cups onto each of the six squares in the grid?

    b. What was the hardest square to end up on?

    c. What was the easiest square?



2. Next explore a 3 x 3 or 4 x 3 grid.

    a. Can you find a way to stack the cups on each of the squares on these larger grids?

    b. Are there any squares that you think are impossible to stack the cups on?

3. What other size grids would you like to explore?

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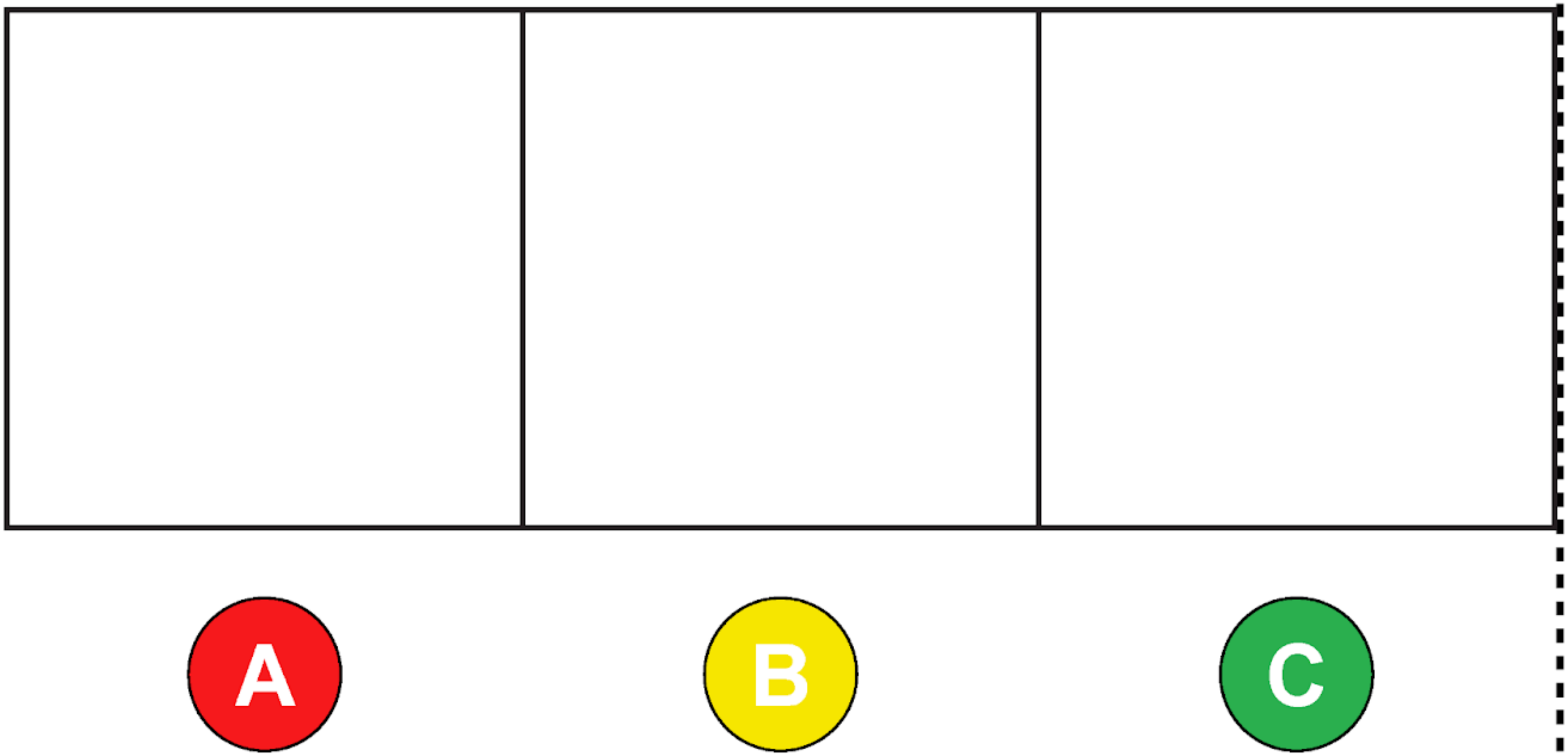

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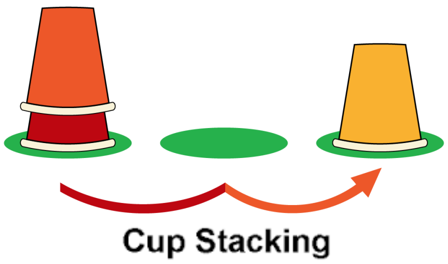
E

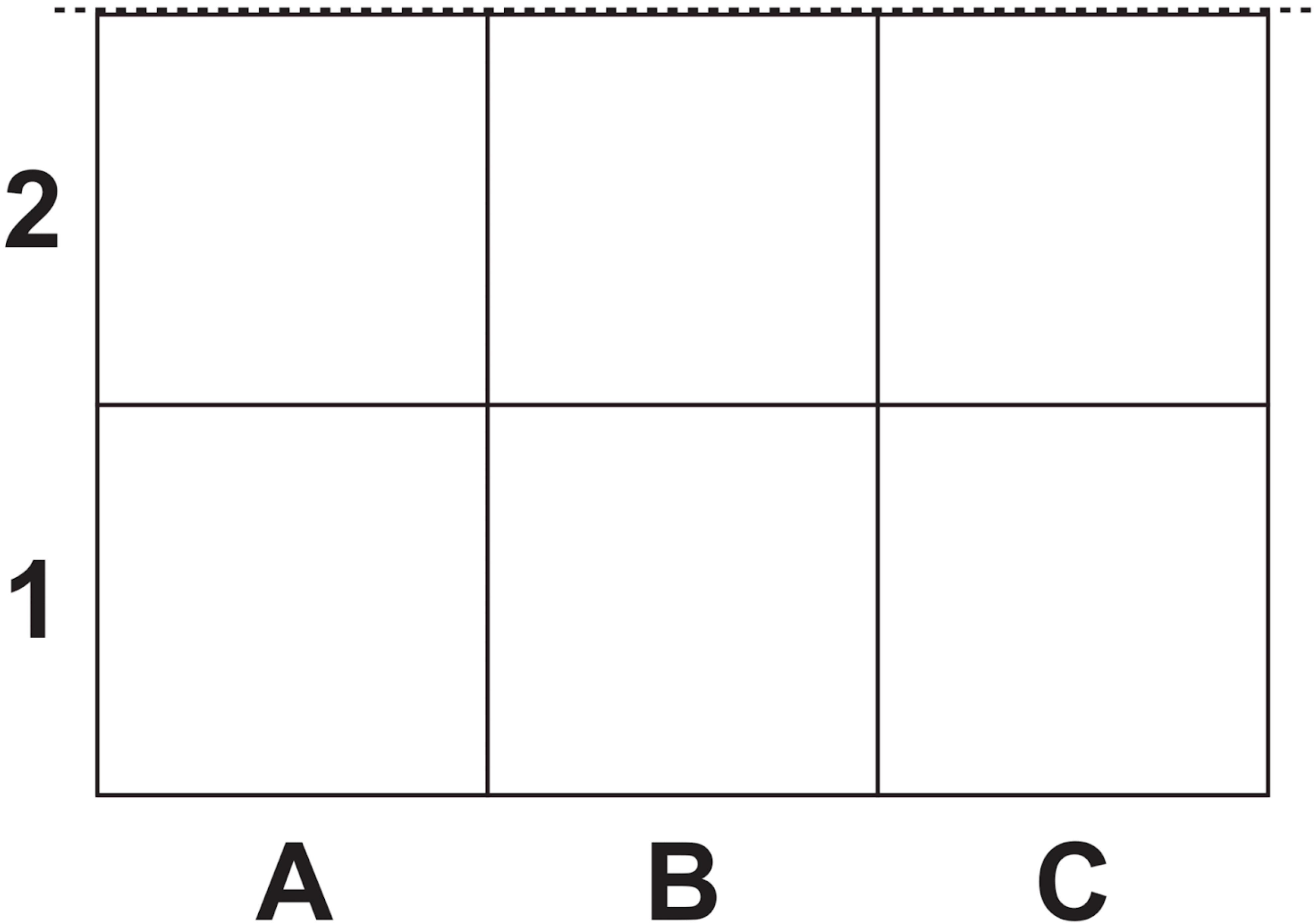
F



G

H

I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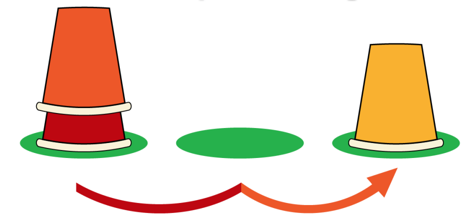
Cup Stacking



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