

CUP STACKING

FESTIVAL GUIDE

TABLE OF CONTENTS

Materials and Setup (p. 2)

Activity Leader Guide (p. 3-6)

Student Instructions (p. 7)

Cup Stacking Tasks (p. 8-9)

Cup Stacking Play Mats (p. 10-12)

Table Sign (p. 13)



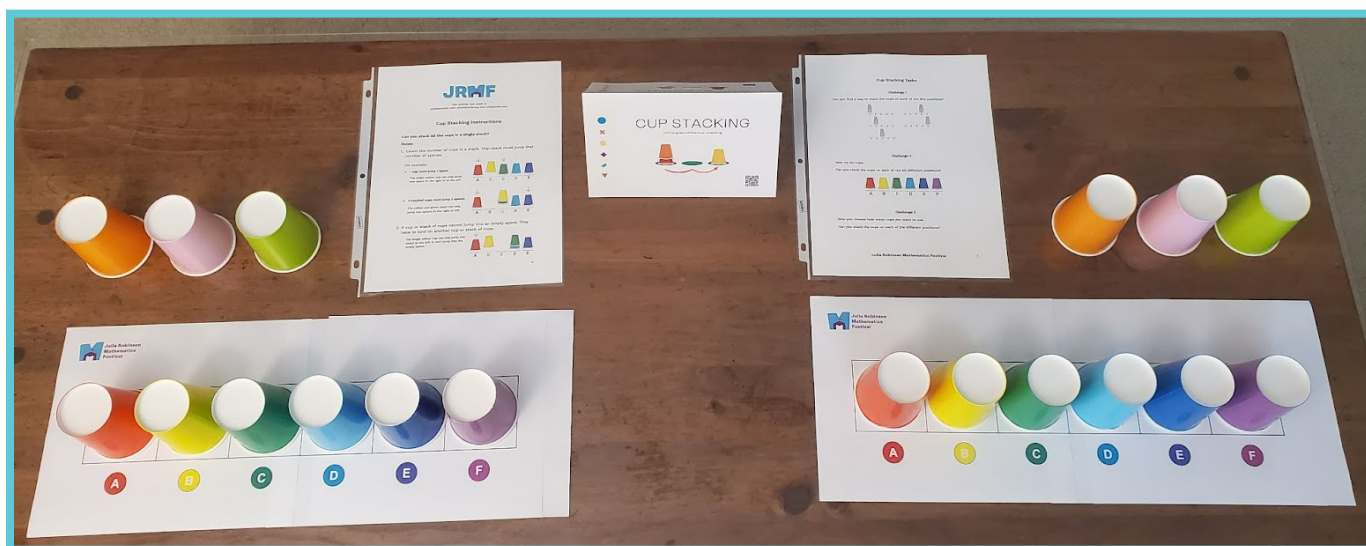
**Julia Robinson
Mathematics
Festival**

Materials and Setup

Per table (assuming 5 students per table), you will need:

Per Table	Material Preparation	
45 small colored cups	Prepare five sets of 9 uniquely colored cups	
3 copies of Instructions	1-page sheet	p. 7
5 copies of Tasks	2-page sheet <i>can be printed double-sided</i>	p. 8-9
5 copies of Cup Stacking Play Mats	3-page sheet <i>print single-sided and splice together</i>	p. 10-12
1 copy of Table Sign	1-page sheet <i>print on cardstock for sturdiness</i>	p. 13
Paper and pencils		

Per Table	Purchasing Materials		
45 cups	set of 200 colored for \$12.99		Each set comes with 5 cups in each of 10 different colors.
8 plastic sheet protectors	pack of 100 for \$7.67	pack of 500 for \$26.99	These are recommended in order to protect the table documents that students will be handling.



Objective

Begin with a row of cups and end with all of the cups in a single stack.

Rules:

1. Count the number of cups in a stack. That stack must jump that number of spaces. For example, 1 cup can only move 1 space; 2 cups have to move 2 spaces; 3 cups have to move 3 spaces...
2. A cup or stack of cups cannot move into an empty space. They have to land on another cup or stack of cups.

Materials

Each Cup Stacking table should be prepped for ~5 stations.

Each station needs:

1. 9 colored, stackable cups.
2. Play mats taped together for placing the cups.
3. Cup Stacking instructions.
4. Cup Stacking tasks.
5. Paper and pencils for tracking cup moves.

How to Play

We strongly encourage you to explore the activity yourself ahead of time.

You can try our digital version here: jrmf.org/puzzle/cup-stacking

Introduce the activity without overexplaining it and without telling what strategies students might want to use. As much as possible, avoid giving away answers. Students should be encouraged to explore, experiment, and learn from their mistakes.

1. Have ready to go a set of 6 cups (use fewer cups with young students) laid out on the play mat.
2. Demonstrate the rules by moving some of the cups.
3. Ask the student to finish stacking all the cups in a single stack according to the rules. Encourage them to explain their thinking as they move the cups.
4. If the student gets to a position where it's impossible to stack the rest of the cups, have them explain why it's impossible. Have them start over, or take back their last move, and see if they can make a different choice.
5. Be aware that at first, students may have many misconceptions about the rules. Here are some to watch out for:
 - a. students will land on empty spaces.

- b. Students will pick up cups they jump over.
 - c. Students will think they can only move one stack, e.g. they will think that if they start with the red cup, then they can only move the stack with the red cup in it.
6. Make sure to catch these misconceptions early. Once students get the hang of it, they'll create their own play!
 7. Have the student find a way to stack the cups but with the cups ending in a different position each time. Encourage them to try more cups.

Standards

1. Make sense of problems and persevere in solving them. CCSS.MP1
2. Construct viable arguments and critique the reasoning of others. CCSS.MP3
3. Model with mathematics. CCSS.MP4
4. Look for and make use of structure. CCSS.MP7

Asking Good Questions

1. Ask questions about confidence.
 - a. When a student asks you “Is this right?”, instead of saying “yes” or “no” right away, ask them how confident they are in their answer. Here are some examples:
 - i. “Maybe. What do you think? How confident are you?”
 - ii. “On a scale of 1-5, how confident are you in your answer?”
 - b. If a student is not confident in their answer, follow up by asking “What would help you feel more confident in your answer?” or “Why do you not feel confident?” This helps you determine how best to help the student through their explorations.
2. Ask students about choices.
 - a. When a student is stuck or shows you a wrong answer, instead of jumping in and showing the student the correct answer, start by asking about the choices that the student made along the way. Here are some suggested steps to follow:
 - i. Start from the beginning.
 - ii. Ask students to show you what they’ve tried so far.
 - iii. When the student gets to a point where they have different choices, ask the student “What other choices can you make here?”
 - iv. Have the student make a different choice and try to solve the puzzle. This helps the student see that they have the power to make different choices during an activity, and they’ll start to do this on their own in the future.
 - v. If you’re familiar with the puzzle or a particular solution, stop the student only when a different choice will help them get to the solution. This will help them feel successful faster without you giving away too much of the answer.
3. Ask students about strategies.
 - a. If a student is getting into the activity and has been doing it for a while, ask the student if there are any strategies they’ve come up with to help them solve the puzzle or win the game.
 - b. Follow up by asking if they think their strategies will work for all puzzles and/or larger puzzles, more complex puzzles, etc. Have the student explore more complex puzzles to test out their strategies.
 - c. This is a great way to encourage a student to dive deeper into an activity and to start looking for patterns, structure, and proofs.
4. Activity specific questions.
 - a. Does your strategy for stacking cups on one of the spaces help you stack the cups on any of the other spaces? (The goal here is for students to see symmetry in their moves).

Answers

General Answers:

1. For any row of cups, it is possible to end in any position.
 - a. Choose a target ending position.
 - b. Treat the cups to the left of the target as a subpuzzle and end with them in position 1.
 - c. Treat the cups to the right of the target as another subpuzzle and end with them in the last position.
 - d. Jump the stacks in position 1 and the last position onto the target position.
2. For any row of cups, the fewest moves is $n-1$ (one less than the number of cups).

Answers to Challenge 5:

1. The first, third, and fifth stacks are possible to make.
2. The second stack is impossible to make. Here is a possible explanation for why it's impossible:

Try to start with any of the cups. That starting cup would need to land on a cup that's different from the cup that's directly below it in the second stack. There is no cup that can be the starting cup, so the second stack is impossible to make.
3. The fourth stack is impossible to make. Here is a possible explanation for why it's impossible:

Dark blue is on top of purple in the fourth stack. The only way this can happen is if dark blue jumps exactly 1 space onto purple. This can happen at any time regardless of how the other cups move, so start by moving dark blue onto purple. Similarly, red must jump exactly 1 space onto yellow, and this can happen at any time regardless of how the other cups move, so move red onto yellow next. At this point, you can see that there is no way for red and yellow to end up on position F and be on top of the stack, so the fourth stack is impossible to make.

Answer to Challenge 6:

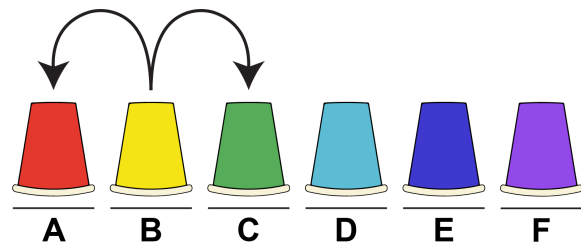
This challenge is only possible if the number of cups you start with is a power of 2.

*This activity was made in
collaboration with sfmathcircle.org and mathpickle.com*

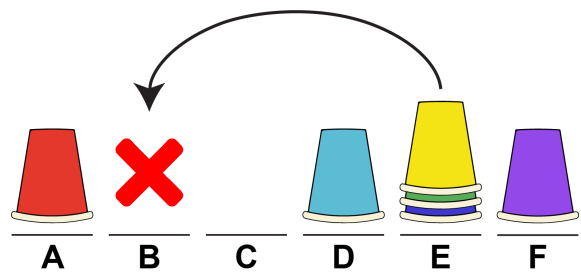
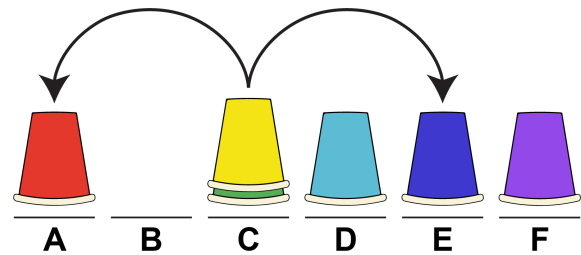
Cup Stacking Instructions

Can you stack all of the cups into a single stack?

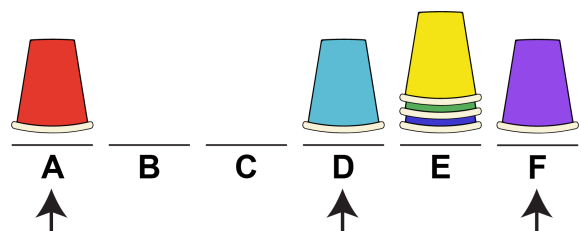
Rules:



1. Count the number of cups in a stack. That stack must jump that number of spaces.



2. Cups cannot land on an empty space.

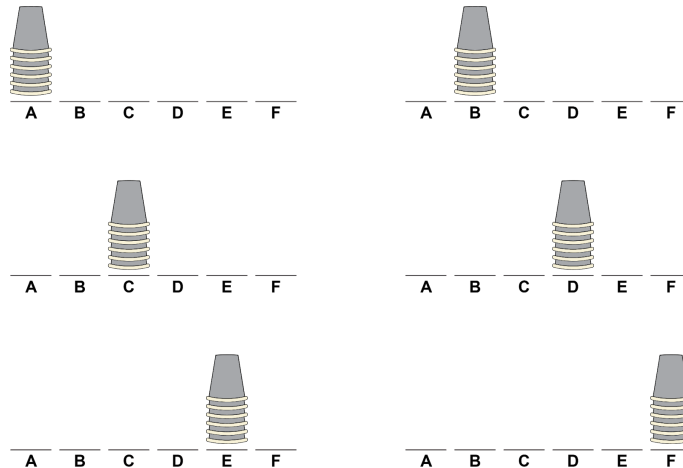


3. You can move any stack you want. How would you finish stacking these cups?

Cup Stacking Tasks

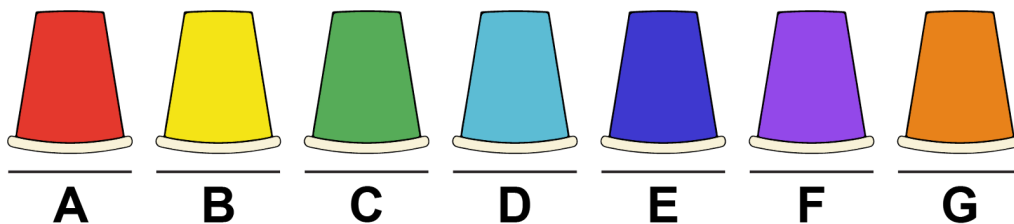
Challenge 1

Can you stack the cups on each of the six positions?



Challenge 2

Now try seven cups. Can you stack the cups on each of the seven different positions?

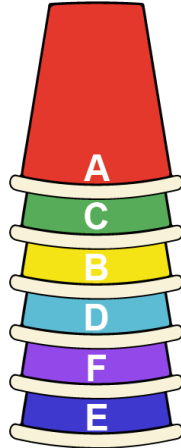


Challenge 3

How about eight cups? More? Can you find a pattern that helps you stack any number of cups quickly?

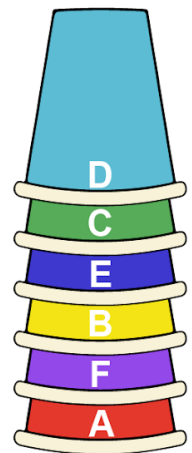
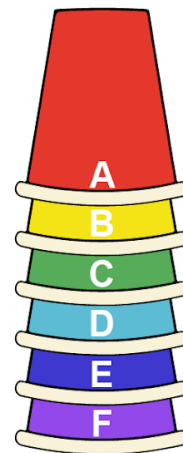
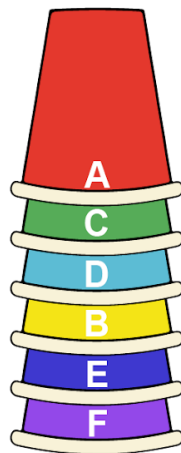
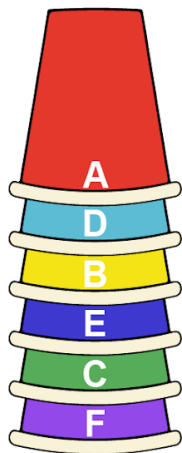
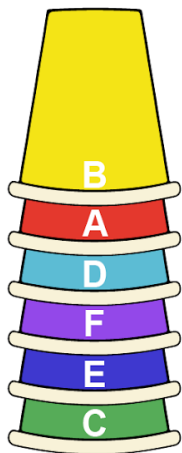
Challenge 4

Set up 6 cups. Try stacking the cups so that the final stack looks like the picture below. It doesn't matter what position the final stack ends up on.



Challenge 5

Try making each of the stacks below. Two of the stacks are impossible to make. Which are impossible, and why are they impossible to make?



Challenge 6

Start with 4 cups. Can you stack all the cups in alphabetical order? If you start with 5 cups, can you stack them all in alphabetical order? 6 cups? More?



**Julia Robinson
Mathematics
Festival**

--	--	--



--	--	--

D

E

F



Cup Stacking

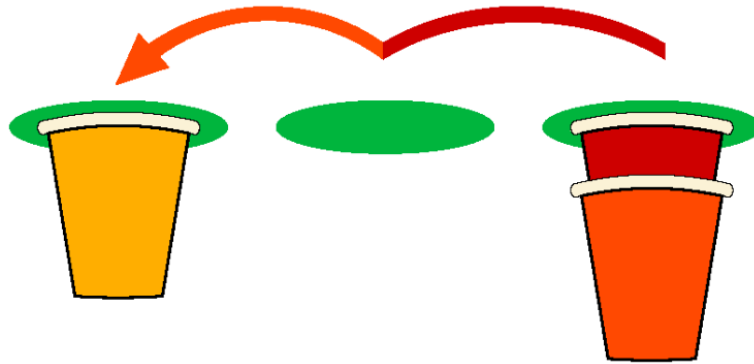


--	--	--





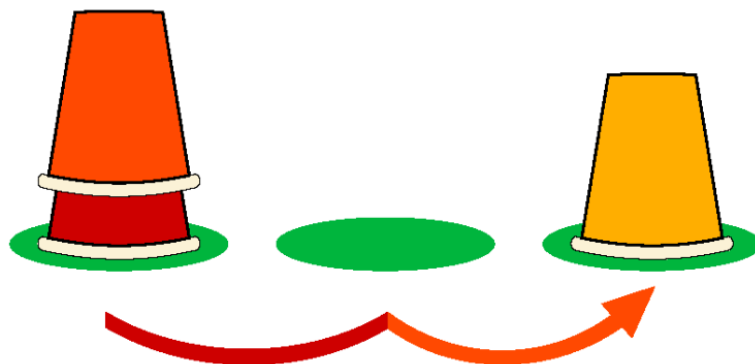
Play for free at
jrmf.org/puzzle/cup-stacking



CUP STACKING



CUP STACKING



Play for free at
jrmf.org/puzzle/cup-stacking

