COUNTDOWN FESTIVAL GUIDE

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Materials and Setup

Per table (assuming 5 pairs of students per table), you will need:

Per Table	Material Preparation					
100 tokens in 5 small containers						
5 copies of Instructions with Tasks	1-page sheet	р. 6				
5 copies of Extensions	1-page sheet (for grades 6-12) print double-sided with above sheet	p. 7				
1 copy of Table Sign	1-page sheet print on cardstock for sturdiness	p. 8				
Paper and pencils						

Per Table			urchasing Materials					
100 two-color counters	<u>pack of 200</u> for \$8.99	<u>pack of 1000</u> for \$37.77	We recommend using the two-color counters as they are useful for other JRMF activities. However you can also use items like rocks, pennies, beans, bingo chips anything you can collect!					
10 plastic sheet protectors	<u>pack of 100</u> for \$7.67	<u>pack of 500</u> for \$26.99	These are recommended in order to protect the documents that children will be handling.					



Countdown Activity Leader Guide

Objective

The goal is to take the last token.

Rules:

- 1. Start with 10 tokens in a pile.
- 2. Players take turns taking 1 or 2 tokens.

Materials

Each Countdown table should be prepped for 5 stations of two students. Each station needs:

- 1. 20 tokens.
- 2. Countdown instructions.
- 3. Countdown tasks.
- 4. Paper and pencils for tracking moves.

How to Play

We strongly encourage you to explore the activity yourself ahead of time.

You can try our digital version here: jrmf.org/puzzle/countdown

Introduce the activity without overexplaining it and without telling what strategies students might want to use. As much as possible, avoid giving away answers. Students should be encouraged to explore, experiment, and learn from their mistakes.

- 1. Demonstrate the rules by playing a game with the student (or pair of students).
- 2. Encourage them to explain their thinking out loud as they choose which move to make.
- 3. Have the student explore the game. Countdown is a 2-player game, so collaborating with a partner is preferred, but the game can also be played with an imaginary partner.

Standards

- 1. Make sense of problems and persevere in solving them. CCSS.MP1
- 2. Construct viable arguments and critique the reasoning of others. CCSS.MP3
- 3. Model with mathematics. CCSS.MP4
- 4. Look for and make use of structure. CCSS.MP7



Asking Good Questions

- 1. Ask questions about confidence.
 - a. When a student asks you "Is this right?", instead of saying "yes" or "no" right away, ask them how confident they are in their answer. Here are some examples:
 - i. "Maybe. What do you think? How confident are you?"
 - ii. "On a scale of 1-5, how confident are you in your answer?"
 - b. If a student is not confident in their answer, follow up by asking "What would help you feel more confident in your answer?" or "Why do you not feel confident?" This helps you determine how best to help the student through their explorations.
- 2. Ask students about choices.
 - a. When a student is stuck or shows you a wrong answer, instead of jumping in and showing the student the correct answer, start by asking about the choices that the student made along the way. Here are some suggested steps to follow:
 - i. Start from the beginning.
 - ii. Ask students to show you what they've tried so far.
 - iii. When the student gets to a point where they have different choices, ask the student "What other choices can you make here?"
 - iv. Have the student make a different choice and try to solve the puzzle. This helps the student see that they have the power to make different choices during an activity, and they'll start to do this on their own in the future.
 - v. If you're familiar with the puzzle or a particular solution, stop the student only when a different choice will help them get to the solution. This will help them feel successful faster without you giving away too much of the answer.
- 3. Ask students about strategies.
 - a. If a student is getting into the activity and has been doing it for a while, ask the student if there are any strategies they've come up with to help them solve the puzzle or win the game.
 - b. Follow up by asking if they think their strategies will work for all puzzles and/or larger puzzles, more complex puzzles, etc. Have the student explore more complex puzzles to test out their strategies.
 - c. This is a great way to encourage a student to dive deeper into an activity and to start looking for patterns, structure, and proofs.
- 4. Activity specific questions.
 - a. What are the numbers of tokens that you like to see left on your turn? What are the numbers of tokens that you don't like to see?

Take 1 or 2 tokens:

- Winning strategy: Always leave the pile a multiple of 3.
- Player 2 has the advantage if the pile starts a number of tokens that is a multiple of 3. Player 1 has the advantage otherwise.

Take 1, 2, 3, ..., n-1, or n tokens:

- Winning strategy: Always leave the number of tokens in the pile a multiple of n+1.
- If the starting number of tokens is a multiple of n+1, then the second player can deploy this strategy. Otherwise, the first player has the advantage.

More generally:

Think of each starting number of tokens as being winning (W) positions or losing (L) positions depending on whether the player who must make a play has the advantage or not. For example, if the player may take 1, 3, or 6 tokens, then it's clear that piles of size 1, 3, or 6 are a W, since the player who must make a move can win the game by taking all tokens. A pile of size 0 is a L, because the player who would move next has just lost.

For other positions, here's a way to decide whether they are winning or losing:

- If a move can be made to turn it into a L, then it is a W.
- If all possible moves lead to a W, then it is a L.

We can decide which positions are which by making a table. Using the example of the game where players may take 1, 3, or 6 tokens, we can start the table:

Tokens	0	1	2	3	4	5	6	7	8	9	10	11	12
W or L	L	W		W			W						

If there are 2 tokens, the only amount that can be taken is 1, which leads to a W, so 2 tokens is an L. For 4 tokens, 1 or 3 can be taken, but both lead to Ws, so 4 tokens is an L. For 5 tokens, by taking 1 token we then arrive at an L, so 5 tokens is a W. We can fill the table out for as many tokens as we want:

Tokens	0	1	2	3	4	5	6	7	8	9	10	11	12
W or L	L	W	L	W	L	W	W	W	W	L	W	L	W

This table provides a winning strategy for one of the players:

- If the number of tokens on your turn is a W, then take any amount that will turn it into an L. We know there's such an amount, since that's how the table was made.
- If the number of tokens on your turn is an L, then the other player will win if playing perfectly, so make a random move and hope they mess up!

These tables will always eventually start to repeat themselves, but it can be tricky to tell when the repetition will start and what the length of the repeated block will be.

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This activity was made in collaboration with sfmathcircle.org

Countdown Instructions and Tasks

How to play:

Take turns taking either **1 token or 2 tokens** from a pile.

Whoever takes the last token wins!

1. Start with 10 tokens and play a few games.

a. Can you find a strategy that helps you win every time?



2. Does your strategy work with more tokens? Fewer tokens?





Countdown Challenges

3. Take 1, 2, or 3 tokens

Play a few games with a 10-token pile, but this time players may take 1, 2, or 3 tokens.

- Which player seems to have the advantage?
- What does the pile look like when you know you're about to win or lose?
- What if you start with 11 tokens? 12 tokens? *n* tokens?

Make a table to track your findings and see if you can explain the pattern you see.

4. Take 1, 2, 3, or more tokens

Play a few games with a 10-token pile, but this time players may take 1, 2, 3, or more tokens.

- Can you find a winning strategy for Player 1 or Player 2?
- Is there a pattern you can generalize?
 - Choose a number *k*. How does this game work if players are allowed to take 1, 2, 3, ..., or *k* tokens from a pile?
 - If there is an *n*-token pile, for which *n* does Player 1 have a winning strategy and for which *n* does Player 2 have one?

Make a table to track your findings and see if you can explain the patterns you see.

5. Take 1, 2, or 4 tokens

Now try playing the game with a 10-token pile but each player can take 1, 2, or 4 tokens -- but not 3 tokens!

- Can you find a winning strategy for Player 1 or Player 2?
- Does the winning strategy change hands for 11 tokens? 12? 13? 14?
- Which player has the winning strategy and what is it on an n-token pile?
- Are the patterns you notice similar to patterns in the previous tasks? Why?



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Play for free at jrmf.org/puzzle/countdown

