Perform Mathematical Magic!

G4G16

How can you spark interest in mathematics? One effective approach is through mathematical magic.

When searching for mathematical magic tricks, I came across a variation of the mind-reading binary number magic trick (mathsbusking.com/shows-mind-reading.html) with the following four cards.

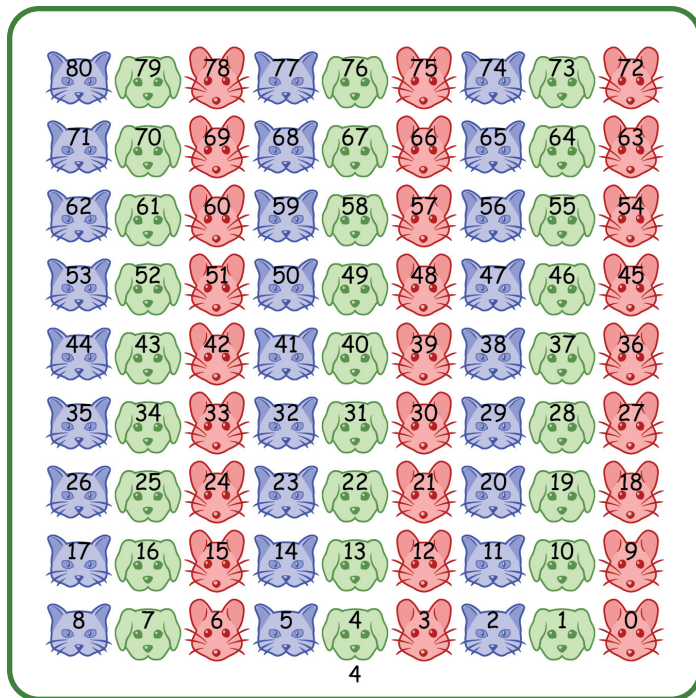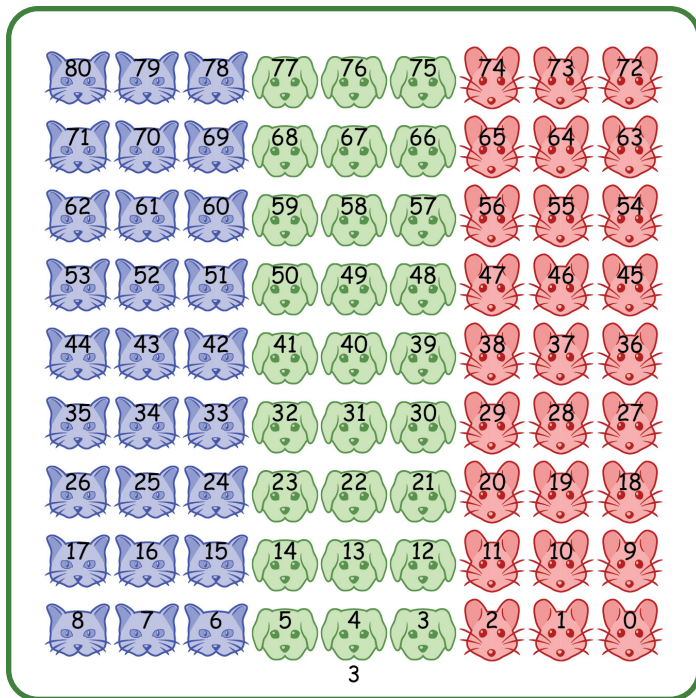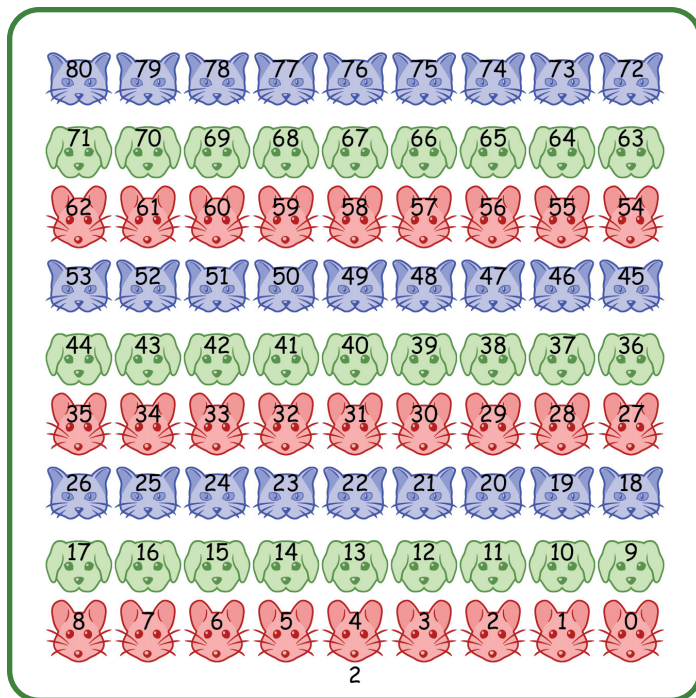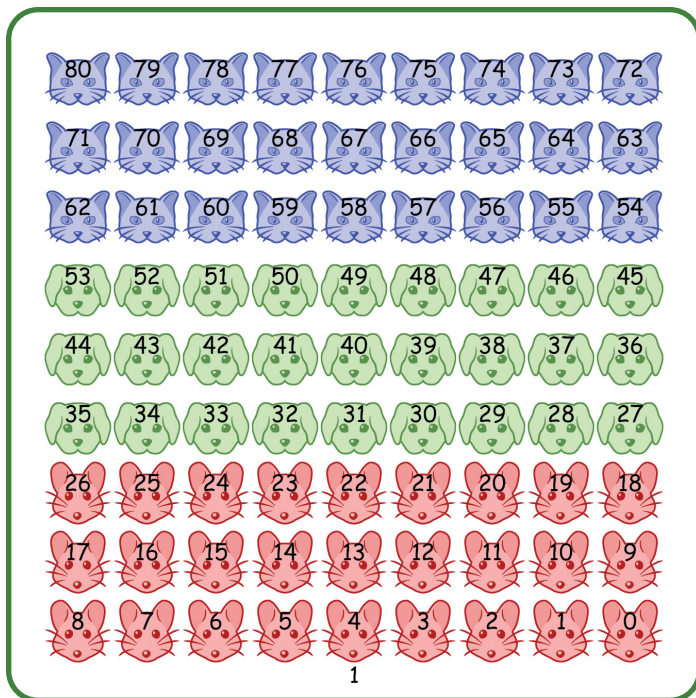To make the trick more engaging, I replaced the geometric shapes with kitties, puppies, and mice. So that it is easier to tell the animals apart, I shaded the animals in different colors. I also modified the layout of the cards (from 8x10 to 9x9) because it's visually appealing to have animals of the same type in straight lines.

Card 1

80	79	78	77	76	75	74	73	72
71	70	69	68	67	66	65	64	63
62	61	60	59	58	57	56	55	54
53	52	51	50	49	48	47	46	45
44	43	42	41	40	39	38	37	36
35	34	33	32	31	30	29	28	27
26	25	24	23	22	21	20	19	18
17	16	15	14	13	12	11	10	9
8	7	6	5	4	3	2	1	0

1

Card 2

80	79	78	77	76	75	74	73	72
71	70	69	68	67	66	65	64	63
62	61	60	59	58	57	56	55	54
53	52	51	50	49	48	47	46	45
44	43	42	41	40	39	38	37	36
35	34	33	32	31	30	29	28	27
26	25	24	23	22	21	20	19	18
17	16	15	14	13	12	11	10	9
8	7	6	5	4	3	2	1	0

2

Card 3

80	79	78	77	76	75	74	73	72
71	70	69	68	67	66	65	64	63
62	61	60	59	58	57	56	55	54
53	52	51	50	49	48	47	46	45
44	43	42	41	40	39	38	37	36
35	34	33	32	31	30	29	28	27
26	25	24	23	22	21	20	19	18
17	16	15	14	13	12	11	10	9
8	7	6	5	4	3	2	1	0

3

Card 4

| 80	79	78	77	76	75	74	73	72
71	70	69	68	67	66	65	64	63
62	61	60	59	58	57	56	55	54
53	52	51	50	49	48	47	46	45
44	43	42	41	40	39	38	37	36
35	34	33	32	31	30	29	28	27
26	25	24	23	22	21	20	19	18
17	16	15	14	13	12	11	10	9
8	7	6	5	4	3	2	1	0

4

Instructions to the Magician

Ask someone, whom I'll call Rowan, to choose a secret number between 0 and 80.

Display card #1 and ask Rowan to tell you the color or animal with the secret number.

Repeat the procedure with the other three cards sequentially, i.e., card #2, card #3, and card #4.

Reveal to the audience Rowan's secret number.
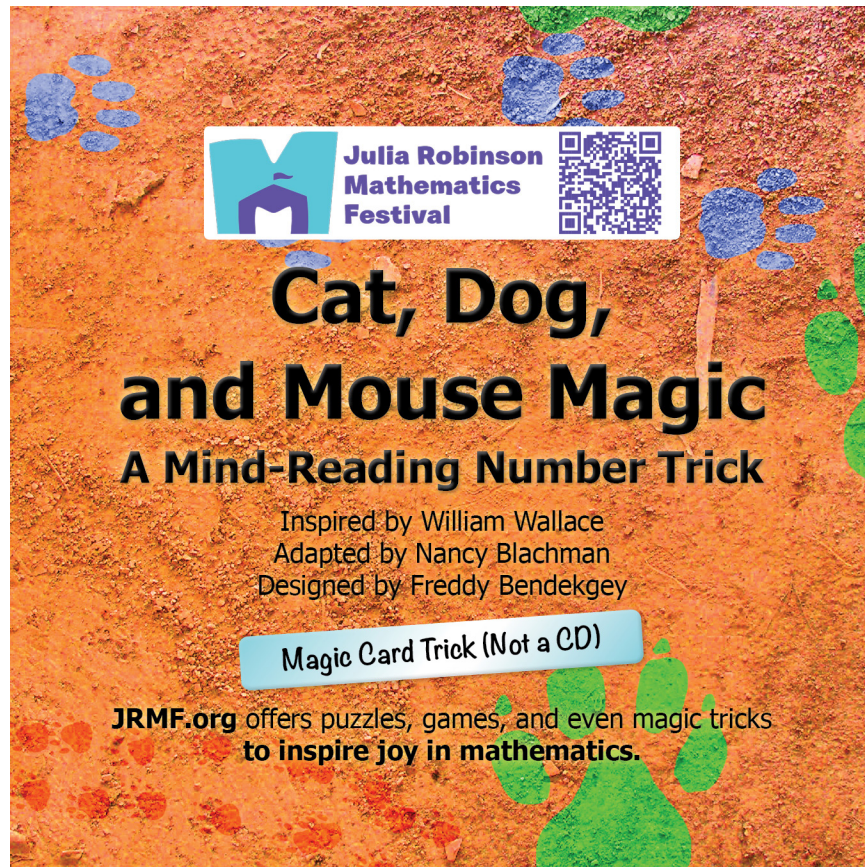
Before you read how the trick works, I encourage you to see if you can figure it out yourself.

How to Perform the Trick

This method for performing the trick works with both versions.

Calculate the secret number as follows: Start with the number 0 in your head. If the volunteer specifies a **blue** cat 🐱 or a **green** dog 🐶, find the smallest **blue** cat 🐱 or **green** dog 🐶 and add the number on that animal to the running total in your head. Don't add anything if a **red** mouse 🐭 (or whatever you call it—mouse or rabbit) is specified. Repeat this procedure for the next three cards. After the 4th card, the sum is the secret number!

Example: Suppose the participant chooses number 42 and doesn't reveal it to the magician.

- On card 1, the number 42 has a **green** dog 🐶 and the smallest **green** dog 🐶 on that card is 27. So start with 27 as the number in your head.
- On card 2, the number 42 has a **green** dog 🐶 and the smallest **green** dog 🐶 on card 2 is 9. So add 9 to the sum in your head to get 36.
- On card 3, the number 42 is a **blue** cat 🐱 and the smallest **blue** cat 🐱 on card 3 is 6. So add 6 to the sum to get 42.
- On card 4, the number 42 is a **red** mouse 🐭. The number in your head remains 42, the secret number!

This trick gives the magician practice in mental arithmetic. After the participant specifies a dog or a cat on card 4, you just need to add 1 or 2 to the sum in your head. Do it quickly and hopefully you'll surprise the participant.

Two versions

I created two versions of the trick. The cards in my original version of the magic trick are numbered 0 - 80 and I packaged them in a black envelope. I put the set of cards numbered 1 - 80 in a green envelope, so it is easy to distinguish the two versions.
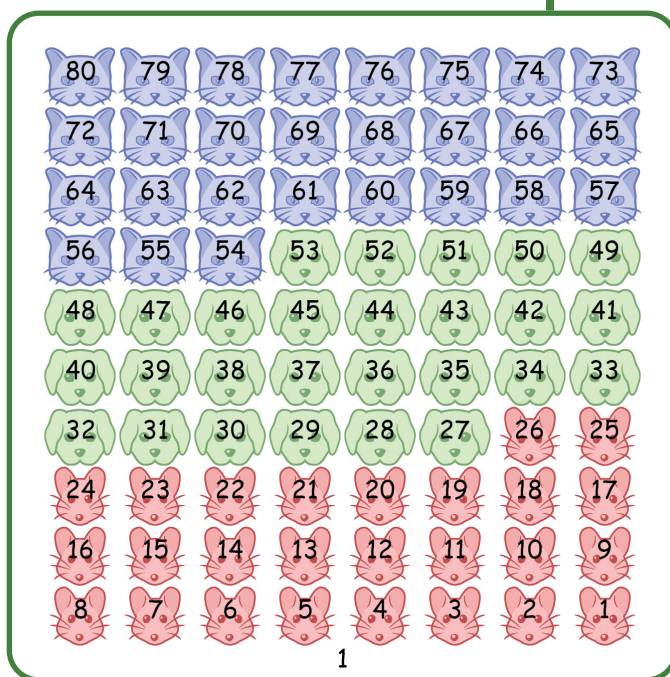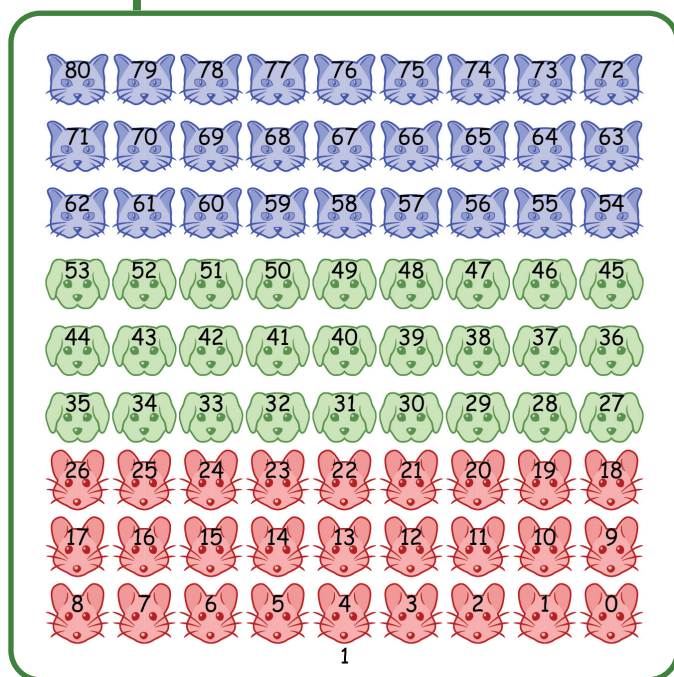
Which version to perform?

- I perform Version 1 (in a black envelope) with numbers 0 - 80 on younger children, i.e., kindergarten - third grade (5-9 years old), because the animals/colors are arranged in patterns, which might enable some to use geometry to figure it out.

- I perform Version 2 (in a green envelope) with numbers 1 - 80 on older children and adults, i.e., fourth grade (9 years old) and older, because it is not as easy to see patterns and the numbers are slightly larger and so more visible for people whose vision isn't great.

Why does the trick work?

Each card secretly represents a place value—just like ones, tens, and hundreds in base 10—but in a different number system. When performing the trick, you are effectively translating the "secret number" from base 3 to base 10.

1. **The secret number:** The animals represent **digits in base 3**:
   - **Mouse** represents the **digit 0**
   - **Dog** represents the **digit 1**
   - **Cat** represents the **digit 2**

2. **Place Values:** The cards correspond to the powers of 3:

   - **Card 1 (3^3):** The **27th place** - On Card #1, all the mice are on numbers that have a 0 in the 3^3 place value (representing the value 0x27=0). All the dogs are on numbers that have a 1 in the 3^3 place value (representing the value 1x27=27). All the cats are on numbers that have a 2 in the 3^3 place value (representing the value 54) .

   - **Card 2 (3^2):** The **9th place** - On Card #2, all the mice are on numbers that have a 0 in the 3^2 place value (representing the value 0x9=0). All the dogs are on numbers that have a 1 in the 3^2 place value (representing the value 1x9=9). All the cats are on numbers that have a 2 in the 3^2 place value (representing the value 18).

   - **Card 3 (3^1):** The **3rd place** - On Card #3, all the mice are on numbers that have a 0 in the 3^1 place value (representing the value 0x3=0). All the dogs are on numbers that have a 1 in the 3^1 place value (representing the value 1x3=6). All the cats are on numbers that have a 2 in the 3^1 place value (representing the value 6).

   - **Card 4 (3^0):** The **1st place** - On Card #4, all the mice are on numbers that have a 0 in the 3^0 place value (representing the value 0x1=0). All the dogs are on numbers that have a 1 in the 3^0 place value (representing the value 1x1=1). All the cats are on numbers that have a 2 in the 3^0 place value (representing the value 2).

3. **The Conversion:** For this trick, the magician converts the four-digit base 3 number to base 10 by summing the following values, which is the smallest number on the specified card that is on the cat or dog that the participant specifies:
   - **Digit** from Card 1 **multiplied** by 3^3 = 27
   - **Digit** from Card 2 **multiplied** by 3^2 = 9
   - **Digit** from Card 3 **multiplied** by 3^1 = 3
   - **Digit** from Card 4 **multiplied** by 3^0 = 1

4. **The Reveal:** Announce the sum. For example, if the secret number is 42, the participant specifies **dog**, **dog**, **cat**, **mouse**, which corresponds to the number 1120 in base 3, sometimes written as 1120_3. Convert the number to base 10 as follows: $1 \times 3^3 + 1 \times 3^2 + 2 \times 3^1 + 0 \times 3^0 = 27 + 9 + 6 + 0 = 42$

6

Resources

I made a video of myself performing the magic trick.

Link: youtube.com/watch?v=EgXx1VPYrgw

Inspired by an AI-generated presentation by Jamie Thompson, I asked him to generate a video with his dog (shown on the right) performing this magic trick. I hope it will amuse viewers and pique their interest.

Link: youtube.com/shorts/pM6oMMnZc4k

After teaching Cat, Dog, and Mouse Magic to 4th graders (typically 9-10 years old), I went to a Christmas holiday party where one of my students went around the party performing the trick on many of the guests, which gave him an opportunity to practice his mathematical skills.

Cat, Dog, and Mouse Magic (Original version, numbers 0 - 80)

•   jrmf.org/puzzle/cat-dog-and-mouse-0-80/

Cat, Dog, and Mouse Magic, Version 2 (numbers 1 - 80)

•   jrmf.org/puzzle/cat-dog-and-mouse-1-80/

Jamie Thompson, with assistance from AI, created a cartoon of a cat narrating version 2 of the trick.

Link: youtube.com/watch?v=6mQDTF1o5zE

Creating Variations

This trick can be extended to other base systems.

How could you change these cards to make a magic trick using a base 4 number system?

A base 5 number system?

A larger-numbered base system?

What animals would you suggest including on those cards for higher bases?

Please share your experience with this trick and/or other mathematical tricks or puzzles with JRMF via their online form jrmf.org/contact-us/.

Nancy Blachman, Founder, Julia Robinson Mathematics Festival jrmf.org

There are two kinds of people: those who love mathematics,
and those who don't yet know they love mathematics.

Booklet designed by Freddy Bendekgey
Mathematical copyediting by Skona Brittain