

## Objective

The goal is to eat the last apple in a row of apples.

Rules:

1. Students take turns eating either one apple or two apples.
2. To show an apple has been eaten, flip over the apple (colored counter).
3. When eating two apples, the two apples must be next to each other.

## Introduction

Tell the students that you're going to show them a game that you're really good at (play it up and call yourself an Apple Master), and that they can get so good at it that they'll be able to be Apple Masters too.

## Explain

1. Lay out a row of 4 or 5 colored counters, red side up and tell the students that these are apples ready to be picked and that the last person to take an apple wins.  
(If you're printing the tasks sheet, start by laying red counters on top of the apples).
2. Tell them they can flip one counter or two counters that are next to each other.

## Engage

1. Ask for a volunteer to play against you and let them decide who goes first.
2. Encourage the volunteer to take suggestions from the rest of the class and to explain their thinking as they choose which move to make.
3. If they lose, reinforce that losing is an opportunity to learn. Could they have made a different choice and won the game?

## Common misconceptions

Students might think that:

- a. They have to start at the far left of the row of apples.
- b. They can flip two apples that are not right next to each other.
- c. They can flip a counter a second time.

## Exploration

In pairs, have your students explore, starting with a row of 4 or 5 apples. Most students start by playing against each other but quickly move to developing strategies with each other.

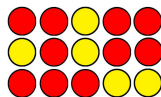
When the pair thinks they have a strategy, have them test it by playing against you or against another pair. If successful, encourage them to explore longer/shorter rows of apples.

Circulate and ask questions to encourage deeper thinking:

- a. If they win: Do you think you can win again? Can you come up with a strategy so that you win every time?
- b. How can you use your knowledge of smaller games to help you create strategies for larger games?
- c. Near the end of a game, how soon can you see who will win?
- d. If the other student has a winning strategy but doesn't make a winning move, can you always win?
- e. When a student is stuck, ask:
  - i. Can you try playing on a shorter row of apples?
  - ii. What have you tried so far?
  - iii. What worked? What didn't work?
  - iv. What are you thinking about trying?
- f. To support pattern recognition, ask:
  - i. What if you track your winning and losing positions?
  - ii. Is there a good starting move?
  - iii. What do you think will happen next?
- g. "Tell me more." is a great basic prompt for getting a student to explain their thinking.

### Extend

1. Play on square or rectangular grids, e.g.,  $3 \times 3$ ;  $4 \times 4$ ;  $2 \times 3$



- a. Students take turns eating 1 apple or 2 apples that are horizontally or vertically next to each other. What's the winning strategy?
2. Have the students come up with their own extensions that they would like to explore, e.g., Play with flipping 1 or 3 consecutive counters, rather than 1 and 2.

### Discussion

As a group, have students share something about their experience with Apple Picking. Try to have at least 3 students share out. Variations of the questions asked earlier are great for generating discussion, such as:

- a. What strategies did you come up with? Did they always work?
- b. What did you try when you first started? What didn't work?
- c. Is there a good first move?
- d. Is anyone sure they can beat me?

## Materials

Two-color counters

Optional: Apple Picking [instructions sheet](#) p. 5

Apple Picking [tasks sheet](#) p. 6

Apple Picking [extensions instructions](#) p. 7

Apple Picking [extensions sheets](#) p. 8

*To explore the activity yourself, you can try our digital version here:*

[jrmf.org/activities/apple-picking](http://jrmf.org/activities/apple-picking)

## Assessment

Evidence of student learning during problem-solving activities can be obtained from three sources: observations, conversations, and products.

**Observation** involves actually observing students while they perform tasks and demonstrate skills and may take the form of a checklist or quick note.

**Conversation** involves engaging students in discussion that encourages them to articulate what they are thinking and then capturing that with a quick note.

**Products** are student-created records that capture not only their answer, but some of the process that led them to the answer.

## Standards

1. Make sense of problems and persevere in solving them.  
CCSS.MP1
2. Construct viable arguments and critique the reasoning of others.  
CCSS.MP3
3. Model with mathematics.  
CCSS.MP4
4. Look for and make use of structure.  
CCSS.MP7

## Answers

### General Answers for Apple Picking:

The key strategy is mirroring.

For the initial game, (counters in a row), Student 1 can always win by using the following strategy:

- a. Student 1 flips the middle counter (if there is an odd number of counters) or the middle two counters (if there is an even number of counters). This separates the game into two equal groups of counters.
- b. Then, for the rest of the game, Student 1 mirrors Student 2's moves across the middle.

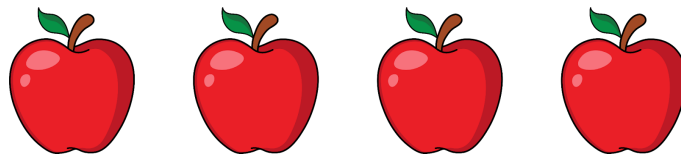
### General Answers for 2D Apple Picking:

1. On a grid of with one or more odd-numbered sides:
  - Student 1 can always win. Student 1 removes the counter or counters from the center, and then mirrors Student 2's move using rotational reflection.
2. On grids with all even-numbered sides:
  - Student 2 can always win by mirroring whatever Student 1 does using rotational reflection. This is because the "center" of the game isn't something that Player 1 can take, unlike in the odd examples. Essentially, Student 1 is in the position that Student 2 is normally in: playing on a game board where the center is missing.

*This activity was made in  
collaboration with [sfmathcircle.org](http://sfmathcircle.org)*

## Apple Picking Instructions

**Can you eat the last apple?**



Take turns eating apples by placing counters on them.

Each turn you can choose to:

Eat 1 apple

OR

Eat 2 apples  
next to each other



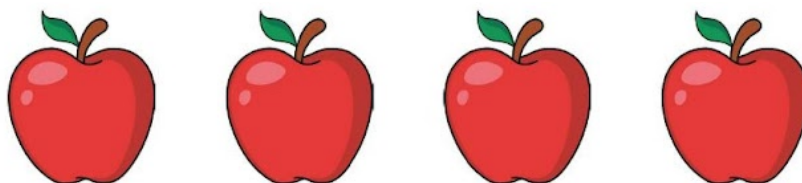
The winner is the player who eats the last apple.

## Apple Picking Tasks

1. Start with one row of 4 apples and play a few games.

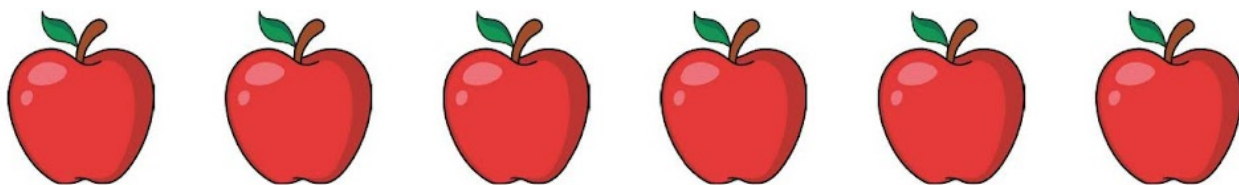
What are some good and bad moves?

Can you find a strategy that helps you win every time?



2. Does your strategy work for longer rows of 5 apples?

Six apples? Seven apples? Even more apples?

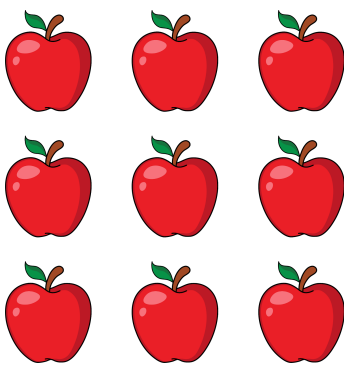




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## 2D Apple Picking Instructions

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Each turn you can choose to:

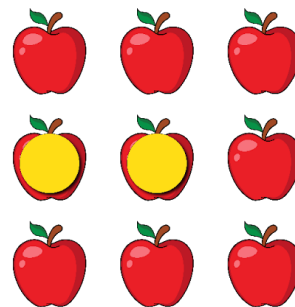
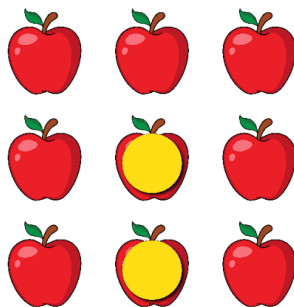
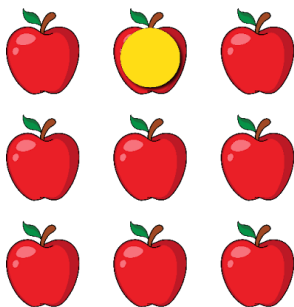
Eat 1 apple

OR

Eat 2 apples on  
top of each other

OR

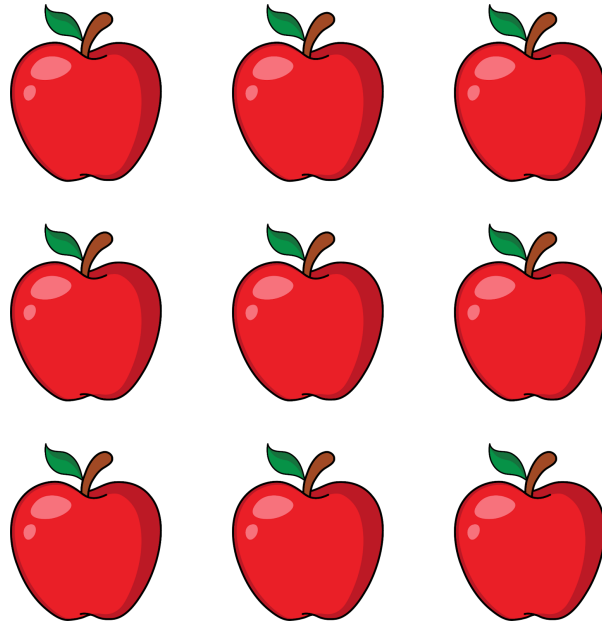
Eat 2 apples  
next to each other



The winner is the player who eats the last apple.

## 2D Apple Picking Tasks

1. Start with a 3x3 grid of apples and play a few games. Can you find a strategy that helps you win every time?



2. Does your strategy work for a 4x4 grid of apples? Larger grids?

