Do you know what happens when you vote?
Is your vote as important as everyone else’s? Can a candidate win the most votes and still lose the election? In this booklet, you’ll explore a common way that politicians decide how much your vote is really worth. It’s called gerrymandering, and the more you know about it, the more power you’ll have when you go to the voting booth.

Happy mathing!

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There are 15 voters in an election. 7 of these voters are going to vote blue (O’s). The other 8 voters are going to vote red (X’s). If all the votes were counted equally, red would win the election. However, that’s not how it works in the U.S. Instead, these 15 voters are grouped into districts — 5 districts with 3 people each.

Red won three districts and blue won two districts, so red won the election! This may not be surprising, since there were more red voters than blue voters. However, can you group these 15 voters in a different way so that blue wins the election?
Can **blue** still win with only **6 blue** voters (and **9 red** voters)? Try it out with the empty districts below.

If there are only **5 blue** voters, there is *no* way for **blue** to win the election. (Why not?) In the election below, there are now 25 voters. You get to choose how many are **red**, how many are **blue**, and how they are grouped. What is the fewest number of **blue** voters that can be part of this election so that **blue** still wins the election?
In real life, politicians don’t get to group voters in any way they want. Districts are based on where people live. Despite this, politicians have still found ways to game the system by choosing districts very creatively. Choosing districts so that one party is made to win (even if that party doesn’t have more votes) is called gerrymandering. Below, there are 25 voters. They are going to be split up into 5 districts with 5 people each. One way to do this is vertically:

Red | Blue
---|---
# of voters: | # of voters: |
15 | 10
# of districts won: | # of districts won: |
5 | 0

Election Winner: Red!

Unsurprisingly, red won the election using the districts above. Even with so many more voters, however, blue can still win the election if the right districts are drawn. Notice that a district is always connected. This means that every square in a district shares a side with at least one other square in that district. As you can see below, this can make gerrymandering hard but not impossible!

Red | Blue
---|---
# of voters: | # of voters: |
15 | 10
# of districts won: | # of districts won: |
2 | 3

Election Winner: Blue!
In each of the voting maps below, there are more red voters than blue voters. Can you gerrymander the maps below so that blue wins each election?
Some maps can be gerrymandered in such an extreme way that practically all of the opponent’s votes seem to disappear! In all of the maps below, blue has fewer voters than red. Despite this, can you gerrymander them so that blue wins four out of the five districts? In the blank map, can you color a voting map that can be extremely gerrymandered in this way?
In each of the maps on this page, blue can win all of the districts, even though there are many red voters in each map. Can you find these extreme gerrymanderings? In the blank map, can you color a voting map that can be extremely gerrymandered in this way?
There are some maps in which no matter how it’s gerrymandered, blue will never win the election. Sometimes this is because there are too few blue squares on the map, but sometimes it’s because of something else. Gerrymander each of the maps below so that blue wins the election. For one of the maps, this is impossible! Which one is it? Can you explain why it’s impossible?
Can you color a different-looking map with 9 blue squares for which it’s impossible for blue to win, no matter how the map is gerrymandered? Can you color a map with 10 blue squares for which it’s impossible for blue to win? How about one with 11 blue squares?
Not every vote matters in an election. For example, in a district that has five people, only three people are needed to win the district — the other two people, no matter what color they are, are wasted votes. Let’s look at the wasted votes in the examples on pg. 3:

<table>
<thead>
<tr>
<th>District #</th>
<th>Winner</th>
<th>Blue Wasted Votes</th>
<th>Red Wasted Votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Red</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Red</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Red</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Red</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Red</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>
Did either of the two voting maps on the other page lead to a “fair” election? What does a fair election even mean? Although this question doesn’t have an easy answer, looking at wasted votes can give us some insight into what fairness looks like. Can you gerrymander the maps below so that in each election, there is the same total number of blue wasted votes as red wasted votes?
In the 2012 election, the voting map for Colorado looked something like the map below. Colorado is divided into seven districts with 20 squares in each district. Can you gerrymander the map below so that red wins the election? Can you gerrymander it in a different way so that blue wins? Can you gerrymander the map in a third way so that the number of red and blue wasted votes are equal (or as close to equal as possible)?

Source: https://fivethirtyeight.com/features/rig-the-election-with-math/
A new, third party, the Green Party (+’s), is starting to gain in popularity. You start to see maps like the one below. Can you gerrymander this map into 7 districts with 7 people in each district so that blue wins the election? Can you gerrymander it in a different way so that red wins? Can you gerrymander it in a third different way so that green wins?

Can you color your own 7 x 7 map below so that all three parties have at least 16 people represented but one of the three parties can’t win the election, no matter how the districts are drawn?
Gerrymandering:
The Game

Objective:
Win the most districts.

Rules:
1. One player chooses to be the Red Party, and the other chooses to be the Blue Player.
2. Using the maps on the next page, players take turns drawing districts with 6 people in each district.
3. When a district is drawn, the player whose party wins that district gains a point (even if that player didn’t draw the district!).
4. A player must draw a district if possible.
5. If a player cannot draw a district that is connected and contains the right number of people, the game is over.
6. There may be squares that do not belong to any district by the end of the game. These squares do not count for any points.
7. The player with the most points at the end of the game wins!

Variants:
1. Create your own maps on the bottom of the next page, and play the same game on your own maps! To be as fair as possible, before the game starts, players take turns filling in the blank maps one square at a time.
2. Play the same game, but the player whose party wins the most districts loses. In this version, if a player can draw a district in which his or her party wins, then he or she must draw that district.
At MAA MathFest in August 2019, I attended a workshop on the Mathematics of Gerrymandering during which we played a game involving drawing boundaries for voting districts so that the candidate who had fewer followers would win the election. Finding the game both engaging and enlightening, I invited Rachel Schmitz, one of the workshop leaders, to facilitate the Gerrymandering activity at the Julia Robinson Mathematics Festival at MathFest. The activity was popular and so we have added it to the JRMF library of activities and created this JRMF booklet.

I wish to thank Kimberly Corum, Towson University, Sand Spitzer, James Rutter, Haystack Mountain School of Crafts, Alexandria Wilhelm, and Rachel Schmitz for presenting The Mathematics of Gerrymandering: Engaging and Authentic Tasks with Civic Significance at MAA MathFest 2019. I also wish to thank Joe Garofalo and Glen Bull, both affiliated with the University of Virginia, for developing the Gerrymandering game kit that was used during the workshop.

Nancy Blachman
Founder, Julia Robinson Mathematics Festival

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