Puzzles with Squares, Cubes, and a Few Other Shapes

Wordless Puzzles

Building a Pyramid

Folding a Cube

A Puzzling Cube

Squares & Cubes

The Muffin Puzzle

“The JRMF really gets it right. Usually the best parts of mathematics are kept away from the public, as if you needed to be a mathematician to get to the fun stuff! It’s refreshing to see a festival that brings this stuff to light, and in such a relaxed atmosphere. If you’re lucky enough to have a JRMF near you, don’t miss it! It’s the best math party around.”

– Vi Hart, Mathemusician, youtube.com/user/ViHart

Festival activities are designed to open doors to higher mathematics for students in grades K–12. Visit www.JRMF.org for more information about Julia Robinson Mathematics Festivals.

Compiled by Nancy Blachman, Founder, Julia Robinson Mathematics Festival.
Play with and explore the richness and beauty of mathematics through puzzles with squares, cubes, and a few other shapes. For more puzzles, visit the websites on the back cover of this booklet.

We invite you to attend or host your own mathematics festival. The last two pages of this booklet contain information about the Julia Robinson Mathematics Festival and how to organize one. Visit jrmf.org/find-a-festival/ for a list of upcoming Festivals. Email info@jrmf.org if you are interested in hosting your own Julia Robinson Mathematics Festival.

– Nancy Blachman, Founded Julia Robinson Mathematics Festival in 2007

Squares

Check out the video puzzle goo.gl/dohrmb from the Global Math Project called Squares. What question is it asking? Can you solve the puzzle?

EXTENSION: Extensions and some puzzles in this booklet were suggested by James Tanton from the Global Math Project and Mark Saul, Executive Director of the Julia Robinson Mathematics Festival.

How many rectangles can you find in a 2x2 grid of squares? The answer is 9, if you consider squares to be rectangles too.

How many rectangles can you find in a 3x3 grid of squares? A 4x4 grid of squares? A 5x5 grid?

Is there a logical reason as to why the answer is a square number each and every time?
Wordless Puzzles

From the book *Without Words*

This puzzle, which is from the video theglobalmathproject.org/december2015 and is included in the book *Without Words* by James Tanton, and puzzles on the following page, is to figure out what each puzzle is and then answer it.

**EXTENSION:**

Select two dots in a five-by-five grid of squares. Develop a general theory that determines whether or not there exists a path of vertical and horizontal steps that visits each and every cell of the grid exactly once, starting at one dot and ending at the other. Does your approach work for a 5x5x5 cube?
Building a Pyramid

Here is an ‘Egyptian’ pyramid (the shape of the ancient pyramids in Egypt):

It has a square base and four faces that are equilateral triangles. We could ‘unfold’ it like this:

![Unfolded pyramid pattern 1]

But we could also unfold it like this:

![Unfolded pyramid pattern 2]

Make sure you understand how to fold these patterns back up to form a pyramid. Which edges of which triangles get attached to which sides of the square? It may help you to label these edges with the same letter or mark.

Here are some more unfolded patterns, but they aren’t complete. Can you complete them?

![Incomplete unfolded patterns]

Is it possible to complete each in more than one way?

When do you consider two unfolded patterns ‘different’?
When do you consider them the same?
How else could we unfold it? Find as many ways as you can.

Hint: Certainly the unfolded pattern must consist of four triangles and a square. To help you, we’ve drawn the square and one triangle below.

![Complete unfolded patterns]
Folding a Cube

Some arrangements of six squares can be folded into a cube, others can’t. A two-dimensional arrangement that can be folded into a three-dimensional object is called a *net*.

Explain why two of the arrangements on the left are *nets of cubes* and the two unshaded arrangements are not.

Draw the net of the cube on the right without the 7 tabs that can be used to glue the sides of the cube together.

Which of the arrangements of six squares can be folded into a cube and which can’t?

This problem is from the AIMSSEC (African Institute for Mathematical Sciences Schools Enrichment Center). [aimssec.ac.za](http://aimssec.ac.za)

EXTENSION:

Each design that successfully answers the FOLDING A CUBE puzzle is called a *net* for a cube.

How many different nets are there for a triangular pyramid (regular tetrahedron)?

What about a ‘shoebox’ (a rectangular solid with three different edge lengths)?

What about a ‘stick of butter’ (a rectangular solid with two different edge lengths)?

Does every three-dimensional polyhedron have at least one net?

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Reprinted with permission from AIMSSEC. The original activity can be found online at [aiminghigh.aimssec.ac.za/grades-6-to-9-cube-nets/](http://aiminghigh.aimssec.ac.za/grades-6-to-9-cube-nets/).

There is a nice interactive version for desktop computers and tablets online at [illuminations.nctm.org/activity.aspx?id=3544](http://illuminations.nctm.org/activity.aspx?id=3544).
A Puzzling Cube

Here are the six faces of a cube - in no particular order:

[Images of six different patterns]

Here are three views of the cube:

[Images of three different views of a cube]

Can you deduce where the faces are in relation to each other and record them on either of the nets of this cube (shown below)?

[Images of two nets of a cube]

Find *A Puzzling Cube* online at [nrich.maths.org/1140](http://nrich.maths.org/1140) and you can find many more NRICH puzzles, problems, and games online at [nrich.maths.org/](http://nrich.maths.org/).
Squares & Cubes

Look at the following expressions and numbers in this table. What patterns do you see?

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<td>2</td>
<td>2 + 3 + 4</td>
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<td>9</td>
<td></td>
<td>1 + 8</td>
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<td>3</td>
<td>5 + 6 + 7 + 8 + 9</td>
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<td>35</td>
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<td>8 + 27</td>
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<td>4</td>
<td>10 + 11 + 12 + 13 + 14 + 15 + 16</td>
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<td>91</td>
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<td>27 + 64</td>
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<td>17 + 18 + 19 + 20 + 21 + 22 + 23 + 24 + 25</td>
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<td>189</td>
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<td>64 + 125</td>
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<td>6</td>
<td>26 + 27 + 28 + 29 + 30 + 31 + 32 + 33 + 34 + 35 + 36</td>
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<td>341</td>
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<td>125 + 216</td>
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In mathematics, a square number or perfect square is an integer that is the square of an integer; in other words, it is the product of some integer with itself. A cube number or perfect cube is an integer that is the result of multiplying a number three times by itself.

Hint: Where do you see perfect squares and perfect cubes in the table at the top of the page?

Fill in some more entries in this table.
Describe in words the pattern you see.

How would you specify the pattern in algebra?
Use the notation \( n^2 \) and \( n^3 \) where you need them.

What would be in the 20\(^{th} \) row of this table?

<p>| | | |</p>
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Can you prove that the pattern works all the time?

The Muffin Puzzle

invented by recreational mathematician
Alan Frank
and described by
Jeremy Copeland
in the New York Times Numberplay Online Blog
wordplayblogs.nytimes.com/2013/08/19/cake

You have 3 muffins and 5 students. You want to divide the muffins evenly, but no student wants a tiny sliver. What division of the muffins maximizes the smallest piece?

Here are some other questions to consider:
- How would you divide 5 muffins between 3 students?
- How would you divide 6 muffins between 10 students?
- How would you divide 4 muffins between 7 students?

For more of a challenge, try:
- How would you divide 11 muffins between 5 students?
- How would you divide 24 muffins between 11 students?
- How would you divide 15 muffins between 19 students?
- How would you divide 12 muffins between 11 students?

What muffin puzzles do you suggest I include in the next version of this booklet?

Bill Gasarch was intrigued by this muffin problem, worked on it for two years, and then co-authored with several others, including some high school students, a paper, which you can find online at arxiv.org/abs/1709.02452. In his paper, he considers the more general problem of $m$ muffins and $s$ students.
Festival Organizers’ Information

What You Need to Know to Get Started Running Your Own Festival

A Julia Robinson Mathematics Festival offers students advanced and thought-provoking mathematics in a social and cooperative atmosphere. Students choose among several tables offering problem sets, games, or puzzles with mathematical themes. They work as long as they wish, while a facilitator provides support and encouragement. Motivation comes from the social interaction, rather than from any prize, grade, medal, or ranking. Festivals are run locally and supported by a national network. They can address any level of student, from those struggling with mathematics to those soaring in achievement.

What is a Julia Robinson Mathematics Festival?
A Festival is an event at which students play with mathematics. Typically, there are a dozen or more tables, each with a facilitator and a problem set, game, puzzle, or activity. Students play and explore individually or in groups, share insights, and make discoveries. Facilitators elicit logical processes for approaching, exploring, or solving problems. The facilitator strives to ask questions rather than provide suggestions or answers. Success is not measured by the number of problems solved nor students’ speed, but rather by how long students stick with activities and by the breadth and depth of their explorations and insights.

Festival activities are designed to open doors to higher mathematics for K–12 students, doors that are not at the top of the staircase, but right at street level.

Who is the Audience?
Festivals are customized for the audience at hand. Local organizers specify their intended audience, and the JRMF organization helps select problems. We support Festivals for students in grades K–3 (usually with their parents), for students in grades 4–6, for middle school students, and for high school students. Some Festival activities are accessible to students with almost no mathematical background, while others engage students with deep mathematical experience. And there are activities for students in between. The social interaction attracts and motivates all kinds of students.

The local organizers decide whether to target certain grades or a wide band of grades. We support festivals for elementary students only, middle school students only, and middle school/high school students. The greater the grade span, the more challenging the festival can be to host.
Why Host a Math Festival?
First and foremost, a Julia Robinson Mathematics Festival brings engaging and deep mathematical content to students in grades K through 12 (ages 4 – 18). Teachers who have experience as a JRMF facilitator use its ‘hands off’ pedagogical style in their classrooms. Our Festivals engage many types of students, including those who don’t enjoy competition or working under time pressure. A Festival is also a community event, bringing together institutions and organizations as their constituents celebrate mathematics.

What Support is Offered to Local Organizers?
The JRMF organization offers:
- A registration system.
- Advice on seeking local funding and recruiting facilitators.
- Help selecting problem sets from our databank of over 100 activities.
- Copy and logos for advertising, banners, and printed materials.
- Training support for facilitators.

How Much Does a Festival Cost?
We never want finances to be an obstacle to hosting a Festival. The JRMF is a non-profit institution whose mission is to inspire interest in mathematics, creativity, and collaboration among K-12 students. We encourage those who can’t afford the costs to apply for a Festival funding grant.

What Happens After a Festival?
We ask that you provide us feedback. We welcome suggestions for how to improve our Festivals and support the hosting organizations. If you are interested in organizing or hosting a Festival, email us at info@jrmf.org.

We would love for you to join our team!

Contact us for more information:

Founder: Nancy Blachman
Executive Director: Mark Saul
EMAIL: info@jrmf.org
PHONE: 917-796-8697
WEBSITE: www.JRMF.org
For more mathematical puzzles, visit...

**NRICH** promotes the learning of mathematics through problem solving. NRICH provides engaging problems, linked to the curriculum, with support for teachers. (Grades K-12) nrich.maths.org

Cool math problems that are beautiful and thought provoking. Favorite lessons and complex problems. (Grades K-6) mathforlove.com/lesson-plan/

On the NY Times website, Numberplay generally presents mathematical and/or logical puzzles and problems. (Grades 5-Adult) wordplay.blogs.nytimes.com/category/Numberplay

MathsChallenge.net is a website dedicated to the puzzling world of mathematics. (Grades 4-Adult) MathsChallenge.net

Dan Meyer has created problems and videos to inspire students to solve problems. (Grades 4-12) blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story

**Alex Bellos’ Monday Puzzle.** (Grades 5-Adult) www.theguardian.com/sience/series/alex-bellos-monday-puzzle

**The Guardian**

**Gord Hamilton has a passion for getting students to realize that mathematics is beautiful.** (Grades K-12) MathPickle.com

**Wild Maths** is mathematics without bounds. Visitors are free to roam and develop as mathematicians. (Grades K-12) wild.maths.org

While a standard textbook cannot adapt to each individual learner, expii.com was created to do just that. (Grades 5-12) expii.com and expii.com/solve

**Brilliant**’s problems are created by people all over the world. Members learn how to solve problems by engaging in a vibrant community. (Grades 2-Adult) brilliant.org

**Project Euler** offers for free engaging computation problems that will require more than just mathematical insights to solve. (Grades 5-Adult) projecteuler.net

**Math Central** is an award-winning website with investigations for teachers and students. (Grades 7-12) mathcentral.uregina.ca/mp

**G4G** features puzzles, games, magic tricks, and crafts. (Grades K-Adult) celebrationofmind.org/puzzles-games

**Youcubed** at Stanford University. Youcubed’s main goal is to inspire, educate, and empower teachers of mathematics, by providing accessible and practical materials. (Grades K-12) youcubed.stanford.edu/tasks

**Inside Mathematics** A resource for educators passionate about improving students’ mathematics learning and performance. (Grades K-12) insideMathematics.org

**The Grabarchuk family produces puzzles for websites, mobile devices, and books.** (Grades 4-12) GrabarchukPuzzles.com