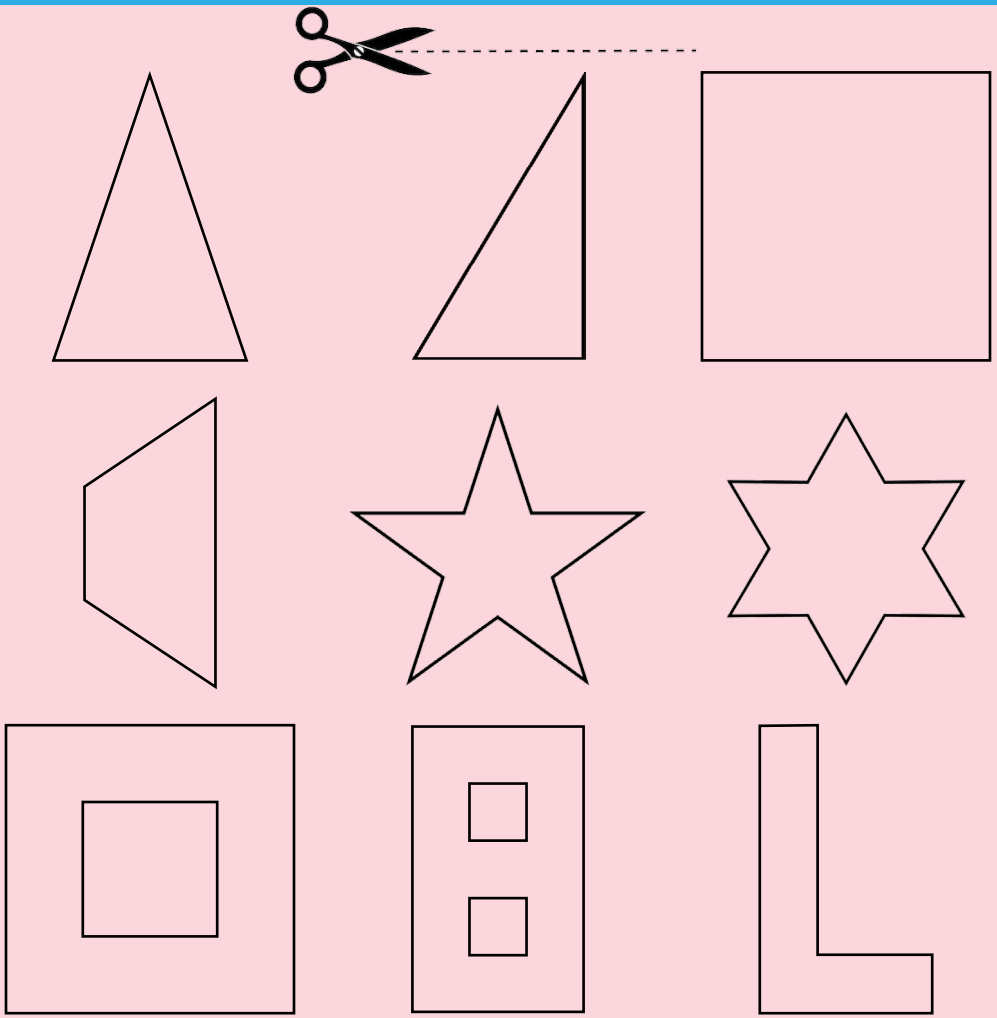
Julia Robinson Mathematics Festival®

Julia Robinson
(1919 - 1985)

Fold-and-Cut Challenge



Festival activities are designed to open doors to higher mathematics for students in grades K–12. Visit **www.JRMF.org** for more information about Julia Robinson Mathematics Festivals.

I was intrigued by the fold-and-cut puzzles that David Klein of the San Francisco Math Circle brought to the Julia Robinson Mathematics Festival at UC Berkeley in the fall of 2018.

After folding and cutting several shapes, I enjoyed reading about paper (folding and) cutting in Chapter 5 of Martin Gardner's book New Mathematical Diversions (MAA, 1995).

> A [...] class of paper-cutting recreation, more familiar to magicians than mathematicians, involves folding a sheet of paper several times, giving it a single straight cut, then opening up one or both of the folded pieces to reveal some sort of surprising result. For example, the unfolded piece may prove to be a regular geometric figure or design, or it may have a hole with such a shape. In 1955 the Ireland Magic Company of Chicago published a small book called Paper Capers, by Gerald M. Loe, which deals almost entirely with such stunts. The book explains how to fold a sheet so that a single cut will produce any desired letter of the alphabet, various types of stars and crosses, and such complex patterns as a circular chain of stars, a star within a star, and so on. An unusual single-cut trick that is familiar to American magicians is known as the bicolor cut. A square of tissue paper, colored red and black to look like an eight-by-eight checkerboard, is folded a certain way, then given a single straight snip. The cut separates the red squares from the black and simultaneously cuts out each individual square. With a sheet of onionskin paper (the thin paper makes it possible to see outlines through several thicknesses) it is not difficult to devise a method for this trick, as well as methods for single-cutting simple geometrical figures.

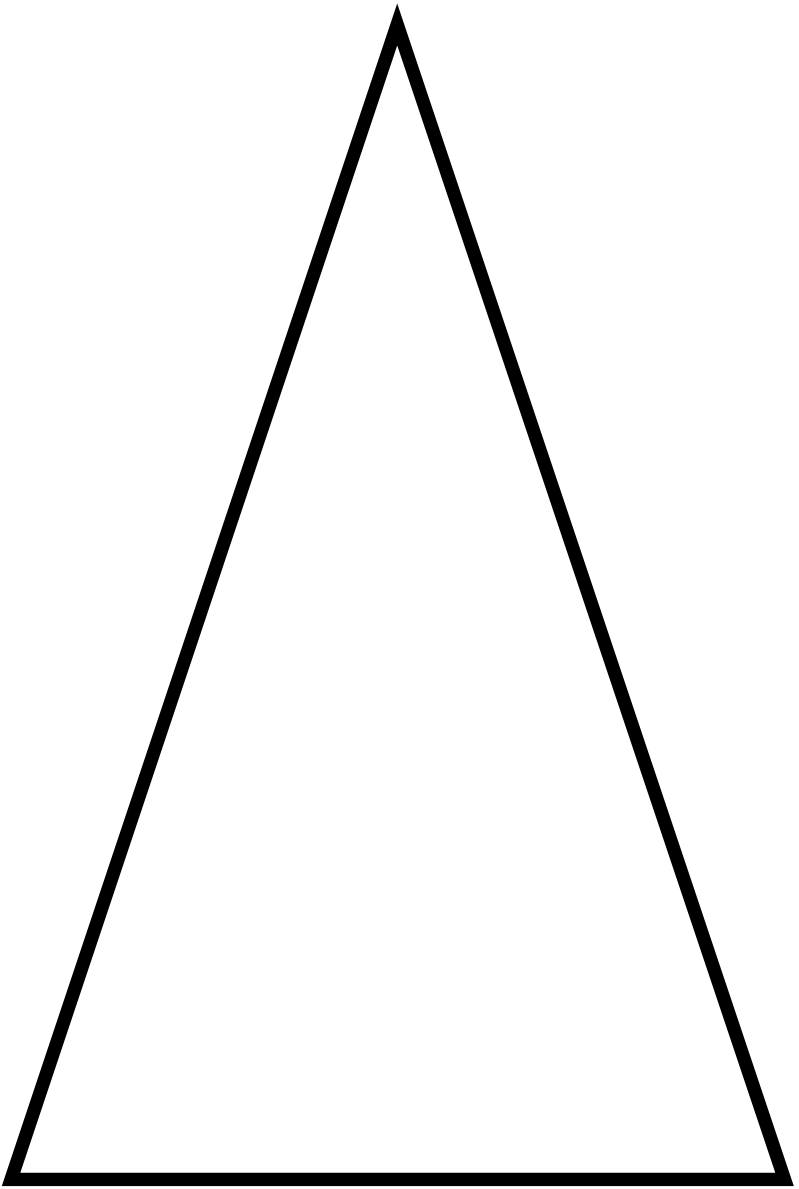
Nancy Blachman
Founder, Julia Robinson Mathematics Festival

Instructions

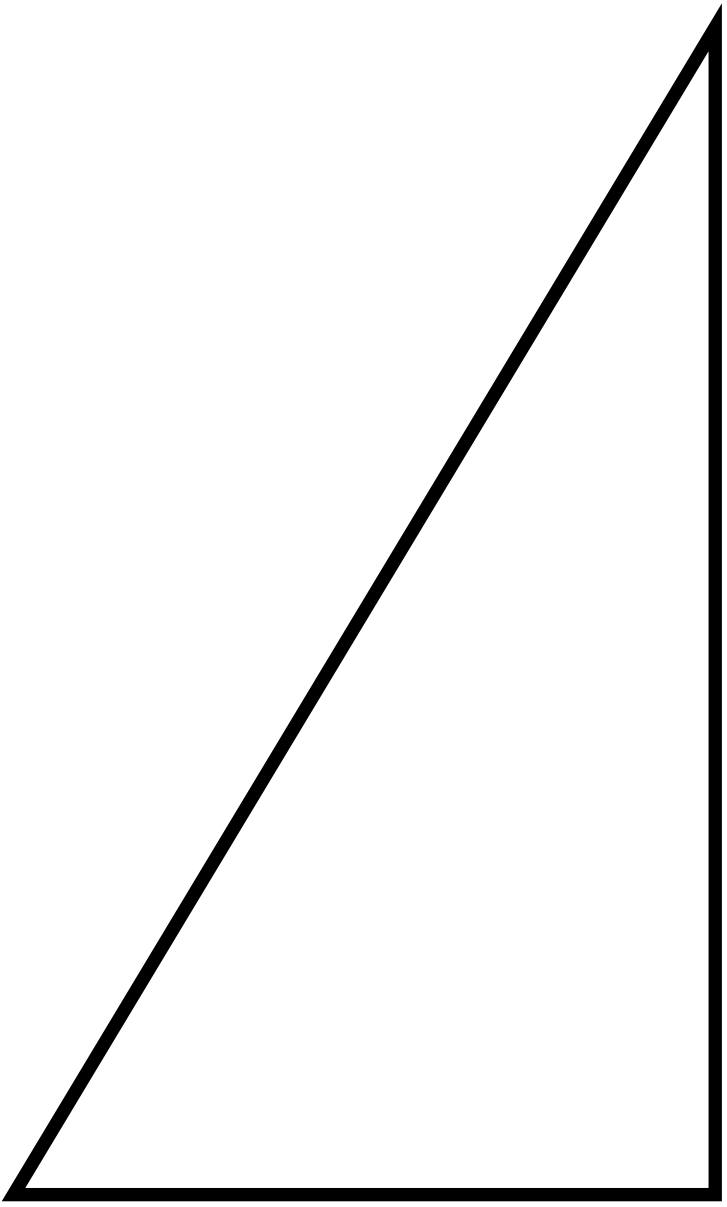
Each page in this book has a different puzzle. To try one of these puzzles, first cut or tear out the page. Then see if you can figure out how to fold the page so that you can cut out the shape with a single straight cut using a pair of scissors (no turning!).

Some of these shapes can be quite challenging! It may take you at least a couple of attempts before you figure out how to cut out a specific shape with a single cut. If you want to print out additional copies of the shapes in this booklet, you can find a file with these shapes (along with even more!) at **bit.ly/2Vb3d4u**.

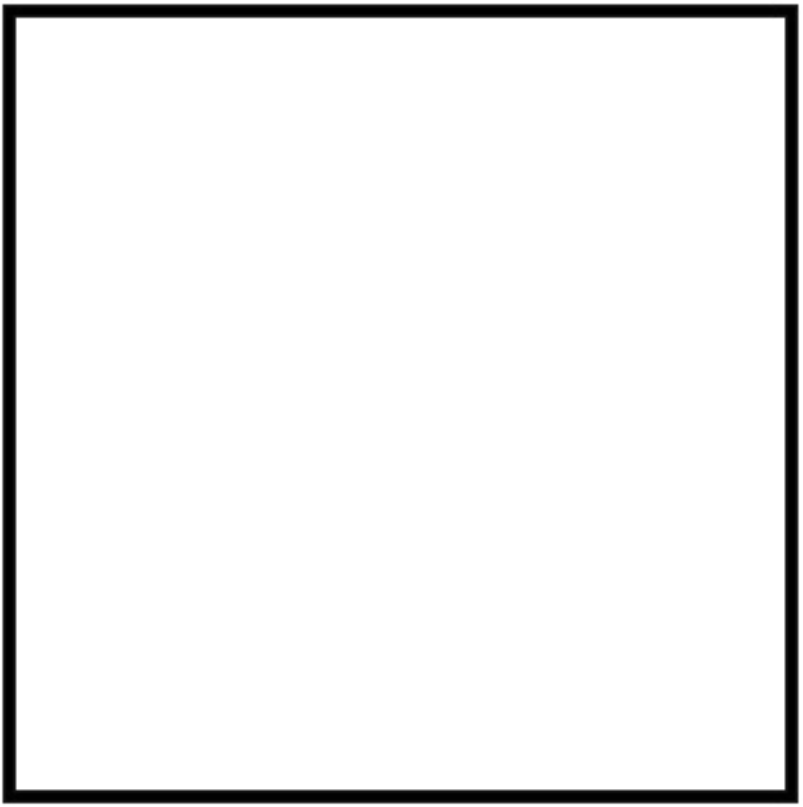
Can you fold this page so that you can cut out this isosceles triangle with a single straight cut?



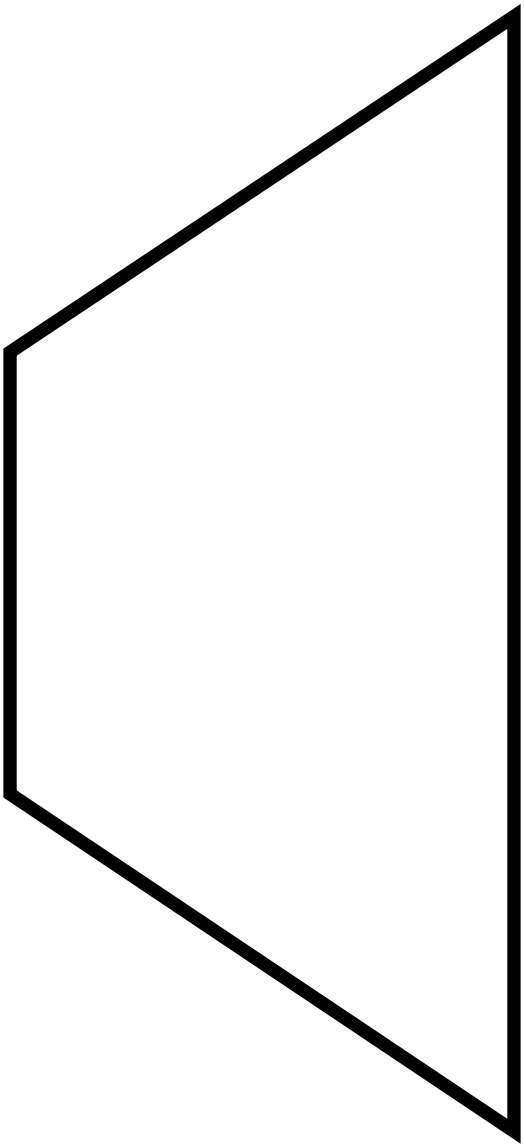
Here's another triangle to cut out. How does this right triangle compare to the isosceles triangle?



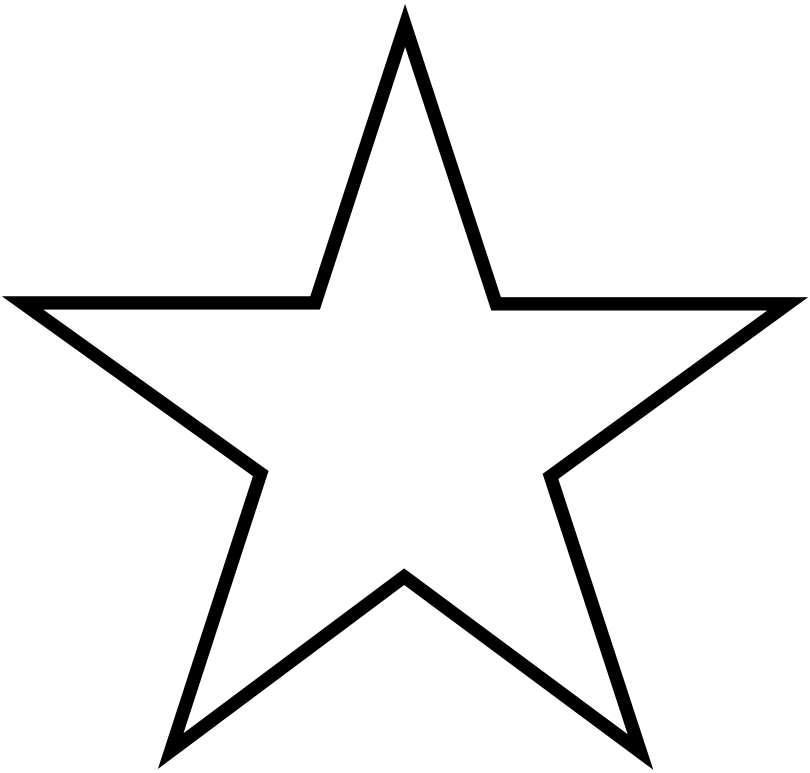
What is the minimum number of folds you need to make in order to cut out this square with a single straight cut?



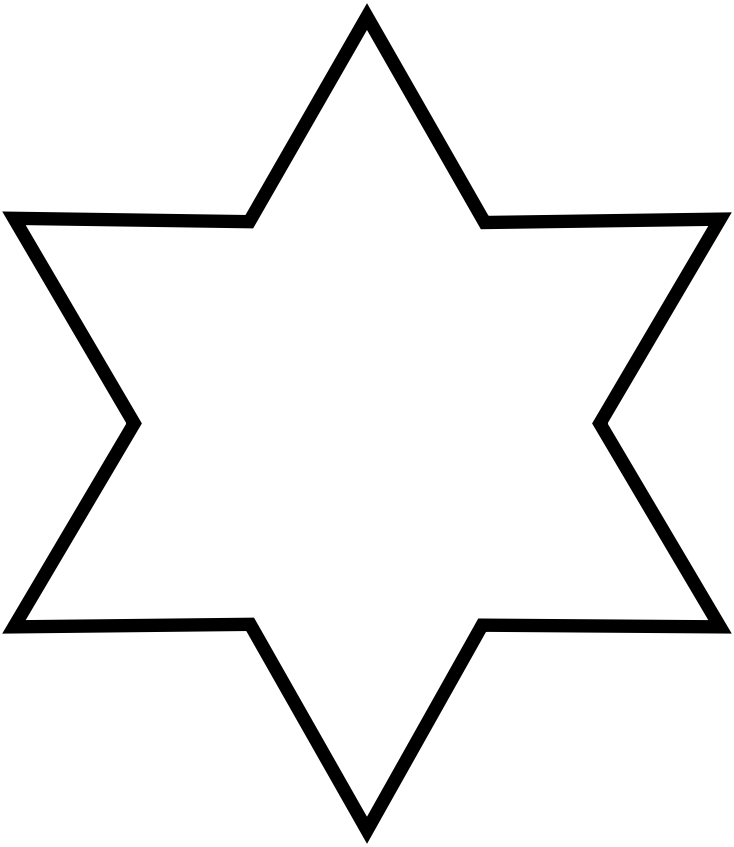
What is your strategy for cutting out this trapezoid? What is your first fold? What next?



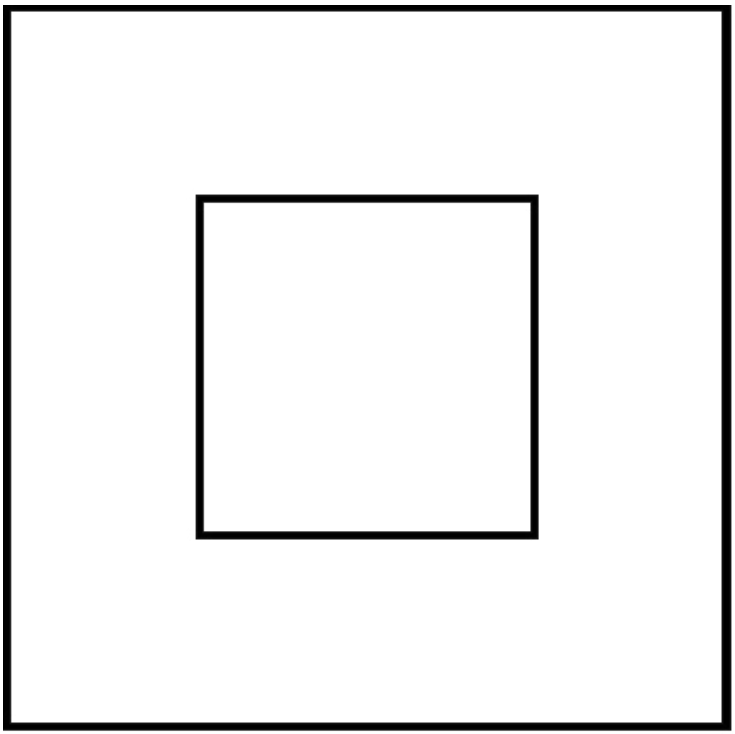
Now on to a stellar shape with ten sides. Can you cut out this five-pointed star with a single straight cut?



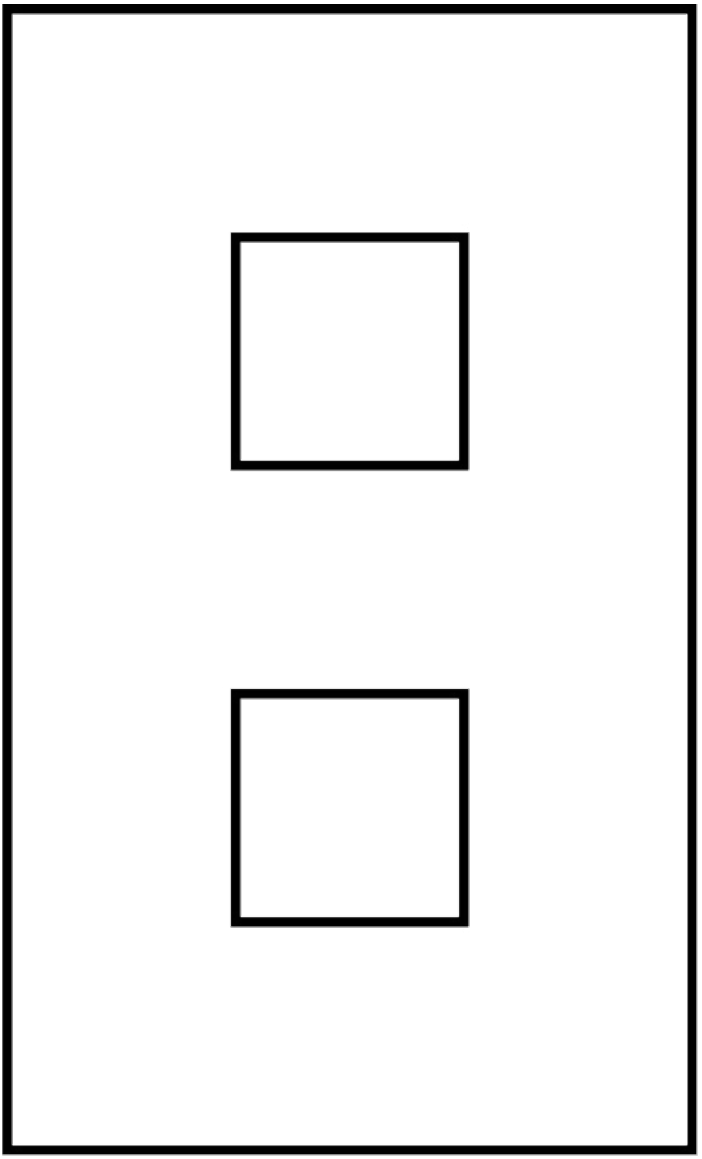
How about this six-pointed star?



Here's a different kind of challenge: Can you cut out **both** of these squares with a single straight cut?

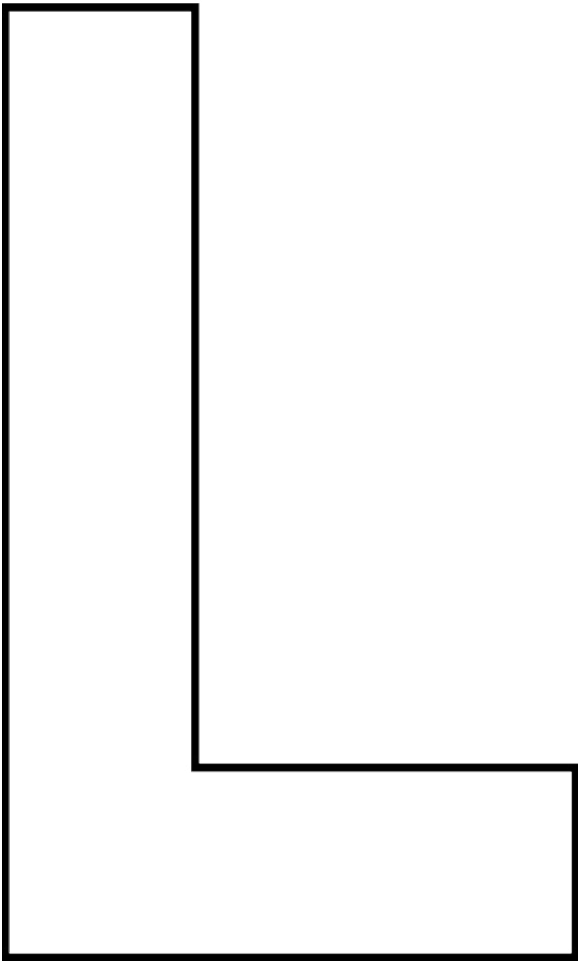


How about both of these squares and the rectangle around them?



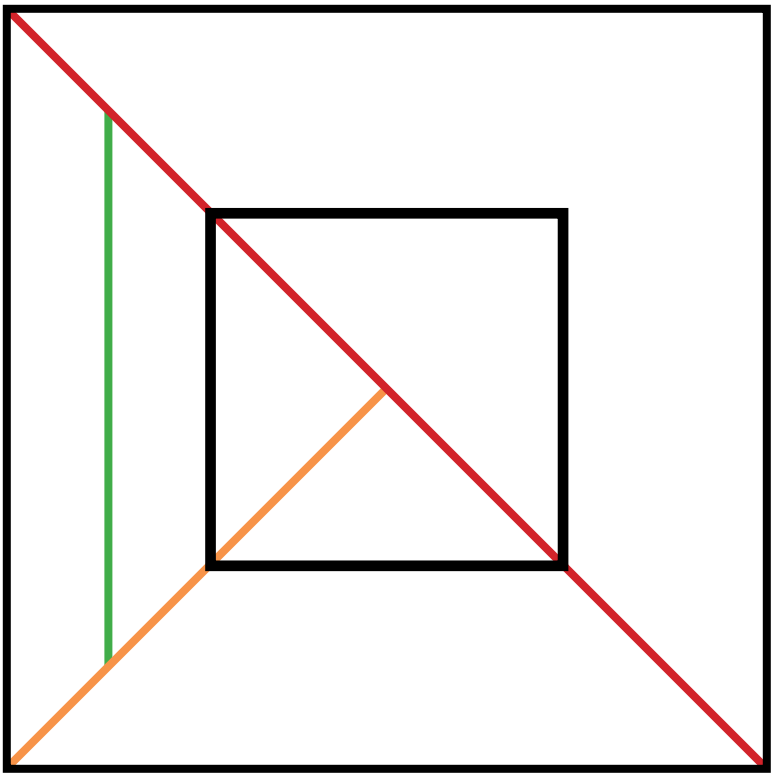
To me, it looks like the number 8. What other numbers can you create with a single straight cut?

What letters of the alphabet can you create by folding a piece of paper and making a single straight cut? Try cutting out this capital L by folding this page and then cutting it with a single straight cut. If you flip the L over, it can be seen as a J (in honor of Julia Robinson).



Experiment and see what other shapes you can make from folding a piece of paper and making a single cut using a pair of scissors. Please share your favorites with us by emailing them to **info@jrmf.org**.

One fun way to demonstrate a solution to a fold-and-one-cut is to make a **rainbow folding diagram**. Here's one for the square within a square:



To cut out this shape:

1. Start by making a mountain fold along the red line.
2. Continue by mountain-folding along each line in ROYGBIV (**red**, **orange**, **yellow**, **green**, **blue**, **indigo**, **violet**) order.
3. When there are no more colored lines, you should only see one black line. Cut along that line!

Can you make rainbow folding diagrams for the other shapes in this booklet?

Related Materials

History of the Fold and Cut Problem - wild.maths.org/history-fold-and-cut-problem

Geometry Journeys - wild.maths.org/interstitial/Geometry-Journeys

Activities inspired by the question: "Is it possible to make any shape with straight sides by folding a piece of paper and making just one straight cut?"

Folding, Cutting and Punching (Activity for ages 7-11), NRICH - nrich.maths.org/1798

Making Maths: Five-point Snowflake (Activity for ages 7-11), NRICH - nrich.maths.org/5374

Erik Demaine's Folding and Unfolding: The Fold-and-Cut Problem - erikdemaine.org/foldcut/

Fold-and-Cut Theorem - en.wikipedia.org/wiki/Fold-and-cut_theorem

Videos

Fold and Cut Theorem - Dr. Katie Steckles, Millennium Mathematics Project - maths.org - www.youtube.com/watch?v=G8SoJ530JAs

Fold and Cut Theorem - Numberphile - www.youtube.com/watch?v=ZREp1mAPKTM

Books

Gardner, Martin. *New Mathematical Diversions*. Washington, DC: The Mathematical Association of America, 1995, Chapter 5: Paper Cutting, pages 58-69.

Loe, Gerald M. *Paper Capers*. Chicago, IL: Magic, 1955.

Though this booklet presents fold-and-cut puzzles, you can also consider two cut-and-fold problems, both involving cubes, posed by Martin Gardner in Chapter 5 of his book, *New Mathematical Diversions*.

1. What is the shortest strip of paper one inch wide that can be folded to make all six sides of a one-inch cube?
2. A square of paper three inches wide is blue on one side and yellow on the other. Rule the square into nine one-inch squares. By cutting only along the ruled lines, is it possible to cut a pattern that will fold along the ruled lines into a cube that is all blue on the outside? The pattern must be a single piece, and no cuts or folds are permitted that are not along the lines that divide the sheet into squares.

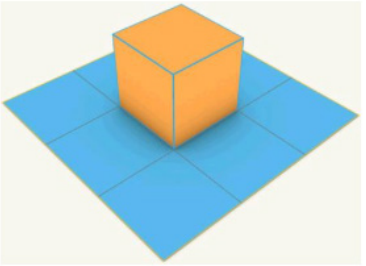


Image: Mathematical Etudes

Julia Robinson Mathematics Festival®

Host Your Own Julia Robinson Mathematics Festival!

A Julia Robinson Mathematics Festival is a non-competitive, extra-curricular event that brings fun, unusual, and advanced mathematics to K-12 students.

A Julia Robinson Mathematics Festival is locally organized. Pick your time, your venue, and your audience, and we will help you run a Festival.

The National JRMF organization will provide:

- Advice from experienced Festival hosts;
- Activities (puzzles, games, problems) tailored to the needs of your audience;
- Publicity and organization items: signs, banners, 'swag' available at cost from our providers;
- Free online registration system.

Photo by Veronica Rosario, Bhakti Seva Art

Our services are offered free of charge to hosts; there might be incidental costs associated with venue rental, materials, and staffing.

You will need to find:

- A suitable venue;
- Facilitators (table leaders) for each activity;
- The participants

. . . AND THE FUN BEGINS!

See **www.jrmf.org** for much more information.

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Executive Director: Mark Saul
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WEBSITE : **www.JRMF.org**

JRMF is a program of the American Institute of Mathematics (AIM).