Julia Robinson Mathematics Festival

Directed By Joshua Zucker

Witch's Cauldrons

You find a witch with two bubbling cauldrons, one filled with bubbling red goop and another with bubbling blue goop. Nearby are frogs, carefully lined up and numbered 1, 2, 3, 4, and so on. The witch instructs you to put frog 1 into a cauldron, and you select the red one. Frog 2, blue. Frog 3, red. Frog 4, blue. Frog 5, red. Frog 6, blue … and suddenly, boom! The cauldron explodes and you are covered in sticky blue goop, as well as three frogs. The witch explains that 2+4 = 6, and so when those three frogs were all in the same cauldron, you get gooped!

1. How many frogs can you place into the cauldrons without getting gooped?

2. Another day, you visit the witch and she has three cauldrons, red, blue, and green. Now how many frogs can you place?

3. What about four cauldrons?

4. Five cauldrons?

5. Another witch has cauldrons that are more difficult to deal with. Now the goop explodes even if you add the same frog's number to itself, so in the original example, the blue cauldron would explode on frog 4, because 2+2 = 4 and frog 2 is already in there! How many frogs can you get into two cauldrons now?

6. Three cauldrons?

7. Four cauldrons?

8. Five cauldrons?

9. Another witch has an even tougher set of cauldrons. If the product of two frogs' numbers is equal to a third frog's number (so you don't need to worry about multiplying by 1), then you get gooped. How many frogs can you get into three cauldrons now?