

Who Can Get the Last Penny?

In each game, there are two players who take turns.
Whoever gets **the last penny wins**.

For each game, your job is to figure out

- How do you know whether it is better to go first or second?
- Once you have made that decision, how can you be sure to win every time?

1. Take One Penny

On each turn, you must take one penny.

- a) Start with 2 pennies on the table.
- b) Start with 3 pennies on the table.
- c) Start with 4 pennies on the table.
- d) Start with 50 pennies on the table.
- e) After counting the pennies on the table, how do you figure out whether you want to go first?

2. Take One or Two Pennies

On each turn, you must take either one penny or two pennies from the pile.

- a) Start with 2 pennies on the table.
- b) Start with 3 pennies on the table.
- c) Start with 4 pennies on the table.
- d) Start with 5 pennies on the table.
- e) You probably will want to try 6, 7, 8 pennies and look carefully for a pattern.
- f) After counting the pennies on the table, how do you decide whether you want to go first?

3. Take One, Two, Three, or Four Pennies

On each turn, you must take either one, two, three, or four pennies from the pile.

- a) Start with 2 pennies on the table.
- b) Start with 3 pennies on the table.
- c) Start with 4 pennies on the table.
- d) Start with 5 pennies on the table.
- e) You probably will want to try 6, 7, 8, or more pennies and look carefully for a pattern.
- f) After counting the pennies on the table, how do you decide whether you want to go first?

4. **Take One Penny, or turn Tails to Heads**

On each turn, you must either take one penny **or** turn one tails penny into a heads penny by flipping it over. You cannot flip a heads penny.

- Start with two heads pennies on the table.
- Start with one heads and one tails penny on the table.
- Start with two tails pennies on the table.
- Try all the different possible starting positions with three pennies.
- What's the best way to play the game when there are 3 heads and 8 tails?
- Can you figure out a pattern that tells you a strategy, so that you can always win this game if you get to decide who goes first?

5. **Take any number of heads, or any number of tails.**

On each turn, you may take as many pennies as you want, as long as they are all the same type as each other (all heads, or all tails). You cannot take a mixture of heads and tails on any one turn.

- Start with two heads pennies on the table.
- Start with one heads and one tails penny on the table.
- Start with two tails pennies on the table.
- Try all the different possible starting positions with three pennies.
- What's the best way to play the game when there are 3 heads and 8 tails?
- Can you figure out a pattern that tells you a strategy so that you can always win this game if you get to decide who goes first?

6. **Take Any Amount Less Than Half**

On each turn, you can take any number of pennies you want from the pile, as long as it is less than (not equal to) half of the number of pennies in the pile. You can always take one penny as long as there's one left to take.

- What should you do when starting with 3 pennies? 5? 6?
- What other numbers can you play and completely figure out, to look for a pattern?
- Can you figure out a rule for how to play with any starting number?

7. **Take Any Amount Up To Double the Previous**

On each turn, you can take any number of pennies you want from the pile, as long as it is less than or equal to **double** the number that the **previous player** took. You can always take one penny as long as there's one left to take, so the first move must be to take 1.

- What should you do when starting with 3 pennies? 4? 5? 6?
- What other numbers can you play and completely figure out, to look for a pattern?
- Can you figure out a rule for how to play with any starting number?