Julia Robinson Mathematics Festival

Directed By Joshua Zucker

Prisoner Puzzle

1. A group of prisoners sits in a circle, numbered from 1 to n. Going around the circle repeatedly, the warden alternately says "skip, cell, skip, cell," and so on, until finally only one person remains. That person is released from the prison. For instance, with 4 prisoners, you skip prisoner 1, send prisoner 2 to the cell, skip prisoner 3, send prisoner 4 to the cell, skip prisoner 1, send prisoner 3 to the cell, and thus prisoner 1 is released.

    a. In fact, on this day prisoner 1 is released, but the total number of prisoners is not 4: it is between 40 and 100. How many prisoners are there?

    b. On another day, when again there are between 40 and 100 prisoners, the very last prisoner (number n) is released. What is the value of n? Is there only one possibility?

    c. On yet another day, prisoner 7 is released. How many prisoners were there that day? Make sure to analyze all possibilities.

    d. Generalize: find a rule for which prisoner number you want to be, if you know how many prisoners will be sitting in the circle.

    e. What if the rule is changed, and instead of the last prisoner being released, it is the second-to-last prisoner who is released? Now what should be your strategy?

2. A new warden is hired whose favorite number is 7. Thus, instead of counting "skip, cell, skip, cell", the new warden counts "1, 2, 3, 4, 5, 6, cell," over and over again, until all but one prisoner has been sent back to the cells. The one remaining prisoner is released. For example, with four people, the warden counts 1, 2, 3, 4, on person number 1, 2, 3, 4. Then the warden continues counting 5, 6, cell, so person 3 is sent to the cell. Continuing, person 4 is sent to the cell next, and then person 1, so number 2 is the desired number.

    a. One day there are 25 prisoners. What seat is the desired one?

    b. If you know how many prisoners there are, find a quick way to decide which number will go free.

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