

## Parity Games

### One-player Games

1. A mad veterinarian (that is, a mad scientist who studies animals) has invented an animal transmogrifying machine. If you put in two cats or two dogs, then one dog comes out of the machine. If you put in one cat and one dog, then one cat comes out. The veterinarian's goal is to end up with only one cat and no other animals. For example, you might start with three cats and a dog.

What happens in this game? Can the veterinarian win?

What if the veterinarian starts with a different collection of animals? For instance, what would you do starting with four cats and two dogs? How about 57 cats and 42 dogs? Can you find a pattern that describes when it is possible for the veterinarian to win?

2. The veterinarian's old machine breaks. Now the veterinarian has dogs, cats, and mice. The new transmogrifying machine can take in any two *different* animals and then out comes the third animal. Can you win (by ending up with just one cat) if you start with three cats and a dog?

What about other starting situations, like four of each animal for example?

What about if you can reverse the rule when you wish, putting in *one* animal and having one of each of the other two animals come out?

3. Begin with some number of coins, say four for example, and set them on the table in a line, with a given starting sequence like HHTH for example. At each move, you may flip any two adjacent coins. You win if the final arrangement of the coins is all heads.

### Two-player Games

4. Player 1 writes a sequence of eight positive integers. Then player 2 writes a + or – sign in each of the seven spaces between the integers. In the end, if the final numeric result is odd, player 1 wins, and if even, player 2 wins. Who should win this game, and how?
5. What if player 1 can only choose numbers (without repetition) from 1 through 10? How about 1 through 9? 1 through 8?
6. What if player 2 must write six + or – signs and one  $\times$  sign?
7. What if player 2 must use exactly two  $\times$  signs (and five + or –)?

8. Two players play a game with one copy of each of the ten digits 0 through 9. Player 1 begins by choosing one of the digits. After that, the players (beginning with player 2) take turns either adding or multiplying the current total by one of the unused digits, until all the digits are used up.

Player 1 wins if the final result is even.

Who should win this game and what strategy can they use?

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