Julia Robinson
Mathematics Festival

Directed by Joshua Zucker
Castilleja School

MySpace Topology

Suppose you are on MySpace and want to see Tom's top 100 friends. To do so, you can click on a link that will give you an option of viewing friends on "View Friends" pages in increments of 20, 40, 60, 80, or 100, where page 0 of the View Friends pages will always be a blank page. For example, if you view friends in increments of 40, page 0 will be a blank page, page 1 will show friends 1–40, page 2 will show friends 41–80, and page 3 will show his remaining friends 81–100. Note that two pages are the *same* if they have exactly the same friends on them. The page number doesn't matter.

Friends

1. How many different View Friends pages are there in total?
2. How many different pages does friend 17 appear on?
3. How many different pages does friend 86 appear on?
4. Which friends appear on the fewest pages?
5. Which friends appear on the most pages?

Intersection and Union

We can think of each View Friends page as an example of a *set* and each friend as an *element* or *member* of that set. For example, the View Friends page that lists friends 1–20 is a set, and since it's a page we might call it P, and to indicate which friends the set contains we could write P_{1-20}.

6. What are the elements of P_{41-60}?

The *intersection* of two sets is the set that contains only the elements that are in both sets.

7. What View Friends page is the intersection of P_{41-60} and P_{1-100}?
8. What View Friends page is the intersection of P_{1-60} and P_{41-80}?
9. The intersection of any number of sets contains only those elements that are in all the sets. What View Friends page is the intersection of P_{1-80}, P_{40-80}, and P_{61-100}?
10. Which friends are in the intersection of all the View Friends pages?
11. Which View Friends page contains the set of friends from the previous question?

12. The *union* of a collection of sets is the set that contains only those elements that are in at least one of the sets. That is, it contains all the elements that are in any of the original sets.

a) Which page is the union of P_{1-20} and P_{21-40}?
b) Which page is the union of P_{1-40} and P_{21-60}?
c) Does any View Friends page represent the set of friends formed by the union of P_{21-40} and P_{81-100}?
d) If we could make new View Friends pages, which friends would appear on a page showing only the friends in the union of P_{21-40} and P_{81-100}?

13. What is the union of all the View Friends pages? That is, which friend(s) if any appear on at least one page?

14. Which View Friends page is the union of all the View Friends pages?

Topology!

While investigating the previous questions you may have noticed that our collection of pages does not include every intersection and union of View Friends pages. What if it did? We call a collection of sets, or of View Friends pages, a *topology* if the collection includes all four of the following:

- Page 0, or in other words, a set containing no elements
- P_{1-100}, or in other words, a set containing all the elements
- the union of any group of pages (sets) in the collection
- the intersection of any group of pages (sets) in the collection.

15. Is Page 0 in the collection we have been discussing?

16. Is P_{1-100} in the collection we have been discussing?

17. Does there exist a group of pages whose union is not in the collection?

18. Does there exist a group of pages whose intersection is not in the collection?

19. Based on the previous questions, why is our collection of View Friends pages not a topology?

20. How many more pages would you need to add to the collection to make it a topology?

21. Using only some of the pages in our collection, what is the smallest topology you can create (has the fewest pages from the collection)?

22. Using only some of the pages in our collection, what is the largest number of pages you can use to form a topology?