

Diophantine Dilemma

1. A store sells pencil lead in packages of 12 or 100.
 - a) Explain why, no matter how many packages of each type you buy, the number of leads you have will always be a multiple of four.
 - b) What is the largest multiple of four that you couldn't buy exactly?
 - c) How many multiples of four (such as 4, 8, and 92) can't be bought exactly?
 - d) Can you represent the patterns in the previous problems in a geometric way?
2. A post office makes only 3 cent and 25 cent stamps.
 - a) What is the largest postage that you couldn't buy exactly?
 - b) How many different amounts of postage could you not buy exactly?
 - c) How does this relate to the previous problem?
3. What if the post office makes only 4 cent and 7 cent stamps?
4. What if the post office makes only 263 and 472 cent stamps?
5. In Klopstockia, they make coins worth only a Klopstocks and b Klopstocks, where a and b are positive integers.
Answer the same questions as in the previous problems, or make your own variations of them.
6. How do the answers to the previous parts change if you are allowed to give change? For instance, with the 4 and 7 cent stamps, you could make 3 cents by buying a 7 cent stamp and giving back a 4 cent stamp.
7. What if there are three different coins? For example, only coins worth 6, 9, and 20. Or a , b , and c ?
8. In the game of Sylver Coinage, players take turns making an integer-valued coin that can't be made out of previously made coins, and whoever makes the coin worth 1 unit loses. For instance, the first player might make 7. Then the next player can't make any coin that could already be made out of existing coins. Maybe they say 10. Then the first player might say 15, the second player might say 3 (it's OK to render a previously made coin redundant), the first player might say 2, and then the second player would have to say 1 and lose. What is a good strategy for this game?