More Cardinality Problems! May 10, 2008

1 Can the plane be a countable union of straight lines?

2 Show that there exists a point X in the plane such that *all* circles centered at X hit at most one lattice point.

3 Show that for each positive integer n, there exists a circle which contains exactly n lattice points.

4 Consider a paper punch that can be centered at any point of the plane and that, when operated, removes from the plane precisely those points whose distance from the center is irrational. How many punches are needed to remove every point?

5 A submarine is traveling exactly one mile below the surface of the water on the number line. At time 0 minutes it was at an integer point, and it travels at a constant integral velocity. Each minute, you can set off an explosive at a single integer point. Without knowing the initial location or speed of the submarine, show how you can destroy it in a finite amount of time.