The Candy Conundrum
Teacher's Guide

The first and most clear goal here is to get students to understand the multiplication counting principle: if you can take some red candies **and** some green ones, then the total number of choices is the product of the number of red choices and the number of green ones.

There's also a fencepost error issue: when do you need to include 0 as a choice, so there are $r+1$ ways to choose from r candies, and when not? The answer to problem 2, for example, is going to have the form $(r + 1)(g + 1) - 1$ because of that.

Then, particularly for middle school, the focus of the early flavor problems is on greatest common divisor, ratio, and proportions.

Next, there's the problem-solving technique of alternative representations. The problem can be numeric, it can be somewhat algebraic or number-theoretic, but it can also be geometric.

As you come to the harder problems, there are a lot more choices of tools. You can use things like the principle of inclusion and exclusion if you know it. But you can also use the strategy of patiently counting some simple special cases and looking for patterns. There's also the strategy of searching online, for example in the online encyclopedia of integer sequences, http://www.research.att.com/~njas/sequences .

For some hints and answers, take a look on the next page!

Selected hints, answers, and solutions

1. **5**. You could take 1, 2, 3, 4, or 5 candies.
2. Here the temptation is to say 5*4 = 20, but actually you can take 0,1,2,3,4, or 5 red candies and 0,1,2,3, or 4 green candies, you just can't take 0 of both. So the number of sets is 6*5–1 = **29**.
3. **119**
4. Only **1**, the pure red flavor. Maybe that was too easy, but it's a good lesson in looking for patterns that you shouldn't skip the easy cases.
5. **17**, I think, but I did it by counting all the possibilities: did I miss one?
6. **94**, but here I'm even less sure.
7. It means they are **not collinear** with the origin.
8. Count the number of different lines through the origin that intersect at least one point with integer coordinates in the rectangle (or box, if there are three flavors). Sometimes it's easier to count, and easier to organize, if you have a picture to draw. But sometimes it's easier to make mistakes, too.
9. Since there's only the point (1,1) on the line of symmetry, there must be an odd number of different flavors.
10. $2^n - 1$
11. 1
12. $3^n - 2^n$
13. I don't know of a simple formula, but see http://www.research.att.com/~njas/sequences/A049691.
14. See the sequences at http://www.research.att.com/~njas/sequences/A090030
15. Let's leave these last problems as open-ended explorations.