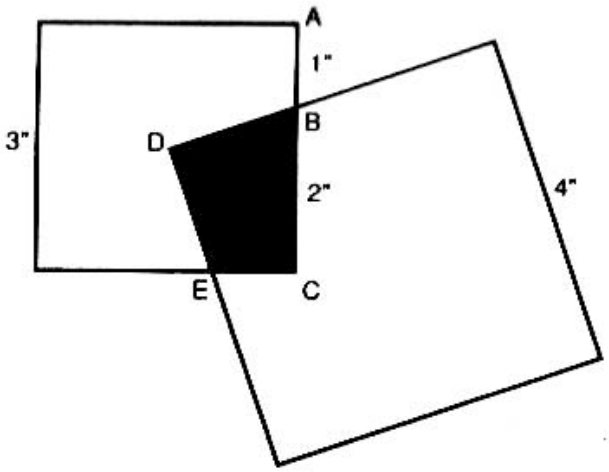
Julia Robinson Mathematics Festival

Directed by Joshua Zucker
Castilleja School

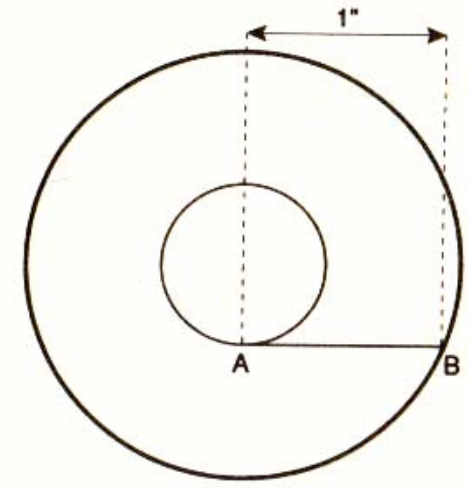
Area Attack!

I. Calculate

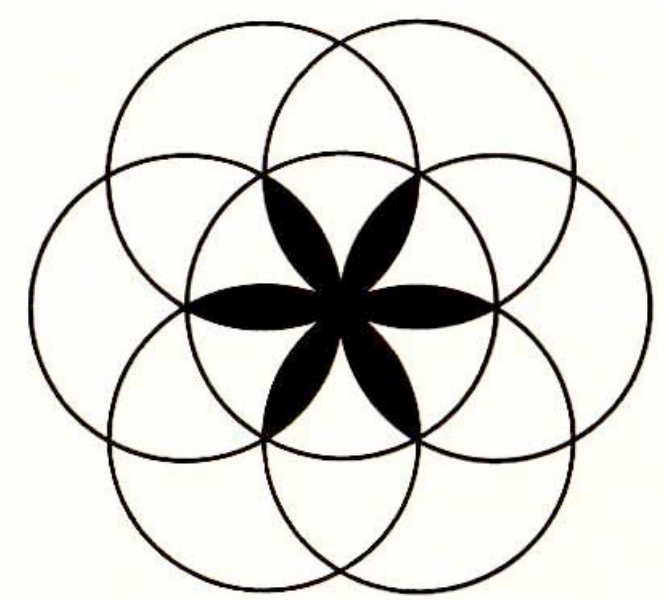
1. The figure below shows two squares. The sides of the smaller square are 3", those of the larger square, 4". Point D is the corner of the large square and the center of the small square. The length of AB is 1" and BC 2". Can you calculate the area of BCED, the overlap of the two squares?



2. In the figure below, line AB, which is 1" long, is tangent to the inner of two concentric circles at A and intersects the outer circle at B. What is the area of the annular region between the circles?

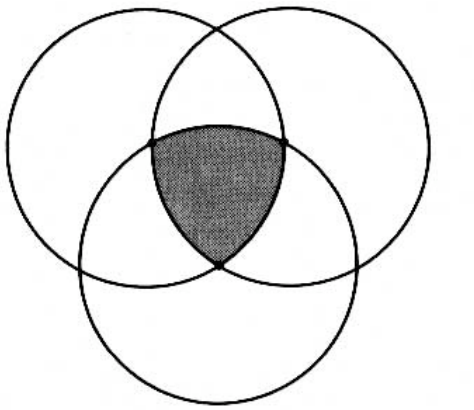


3.

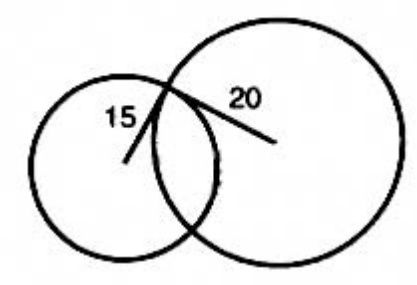


The hex sign above is made up entirely of circles 1" in radius. Can you calculate the area of each of the shaded petal-shaped areas?

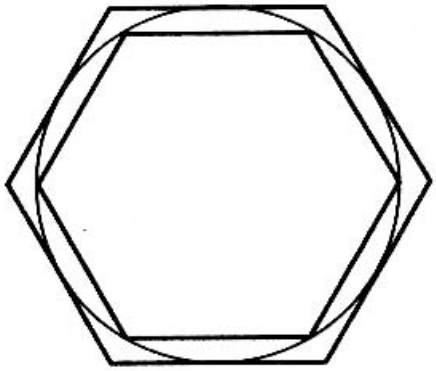
4. In the figure below, 3 equal circles have been drawn so that each one passes through the centers of the other two. Is the area of overlap, shown shaded in the diagram, more or less than a quarter of the area of a circle?



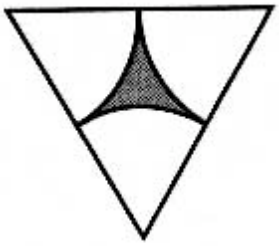
5. A circle of radius 15" intersects another circle, radius 20", at right angles (see below). What is the difference of the areas of the non-overlapping portions?



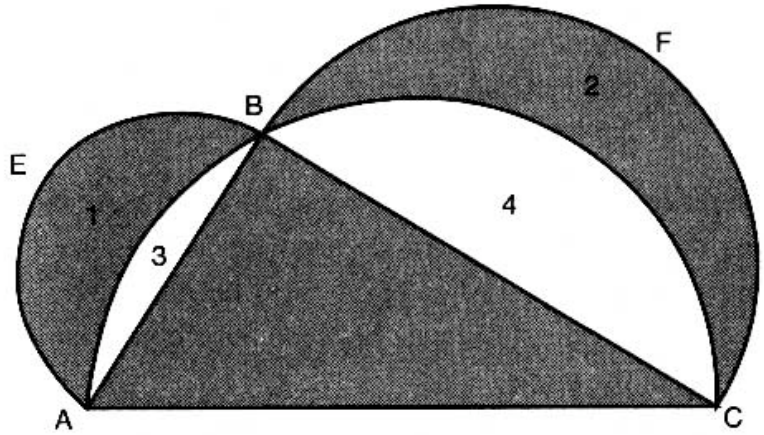
6. Regular hexagons are inscribed in and circumscribed outside a circle, as shown below. If the smaller hexagon has an area of 3 square inches, what is the area of the larger hexagon?



7. The tessellated figure below consists of 3 equal arcs in an equilateral triangle. Each side measures 2". What is the area of the shaded part?

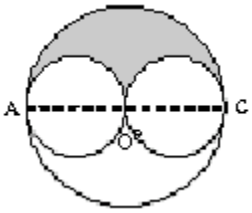


8. Prove that the total area of the regions 1 and 2 below equals the area of the triangle.



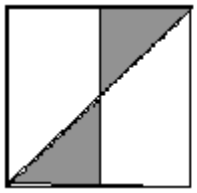
The unshaded area is part of the semi-circle in which the triangle is inscribed, and AB and BC are diameters for the other two semi-circles.

9. **(AJHSME, 1986)**
The larger circle has diameter AC. The two small circles have their centers on AC and just touch at O, the center of the large circle. If each small circle has radius 1, what is the value of the ratio of the area of the shaded region to the area of one of the small circles?

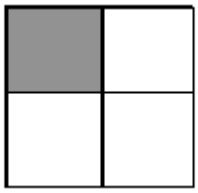


10. **(AJHSME 1994)**
Each of the three large squares shown below is the same size. Segments that intersect the sides of the squares intersect at the midpoints of the sides. How do the shaded areas of these squares compare?

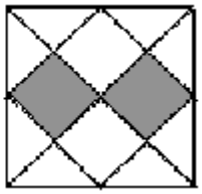
I



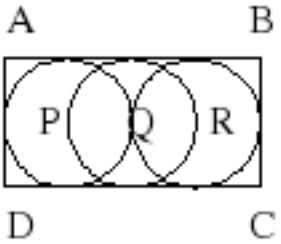
II



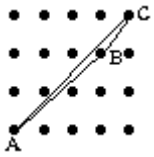
III



11. **(AJHSME 1995)**
Three congruent circles with centers P, Q and R are tangent to the sides of rectangle ABCD as shown. The circle centered at Q has diameter 4 and passes through points P and R. Find the area of the rectangle.



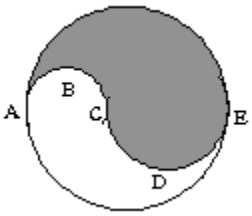
12. **(AJHSME 1996)**



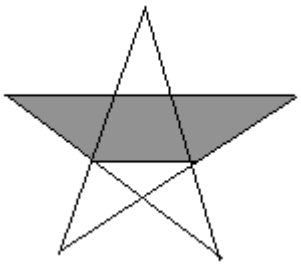
The horizontal and vertical distances between adjacent points equal 1 unit. Find the area of triangle ABC.

13. **(AJHSME 1997)**
What fraction of this square region is shaded? Stripes are equal in width, and the figure is drawn to scale.

14. **(AJHSME 1997)**
Diameter ACE is divided at C in the ratio 2:3. The two semicircles, ABC and CDE, divide the circular region into an upper (shaded) region and a lower region. Find the ratio of the area of the upper region to that of the lower region.

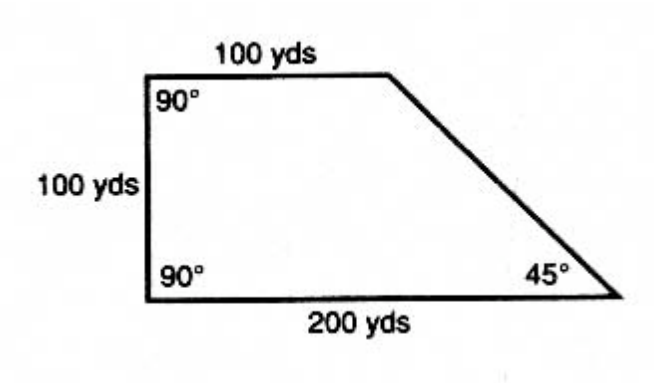


15. Find the area of the shaded region as a fraction of the area of the entire regular pentagram.
Note: Figure not to scale.

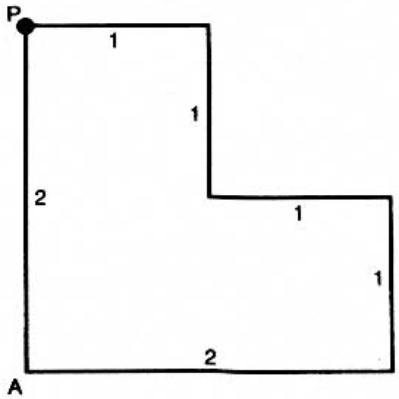


II. Divide

1. A man leaves the piece of land (see below) to his 4 sons, with instructions that it should be divided up into 4 equal pieces, each having the same shape as the original piece of land. How can this be achieved?

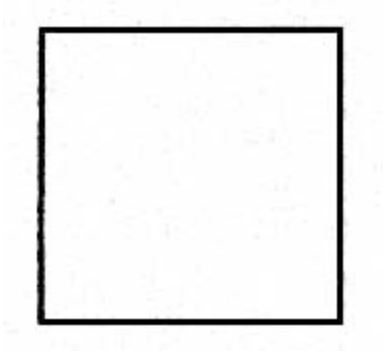


2.

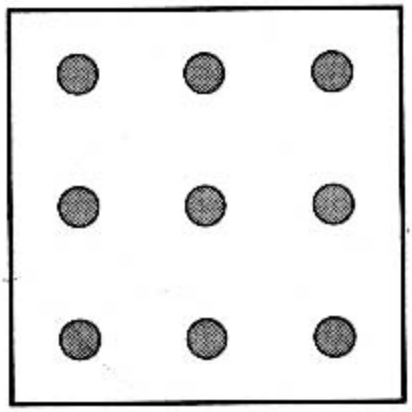


Divide the rectilinear right-angled figure shown above into two equal areas with one straight line PP_1.

3. Using only a straight-edge **and** a compass, construct 5 smaller squares, the areas of which will total that of the larger square.



4. A zoo keeper houses 9 llamas all together in a large square cage.



Strangely, all these llamas are very lethargic and remain in the position shown in the figure above at all times. Can you give each its own private cage by building just 2 more square enclosures?

Acknowledgements

Problems selected, edited, arranged, and some written by Paul Craciunoiu. The first 8 problems are from "The Ultimate Book of Puzzles, Mathematical Diversions, and Brainteasers", by Erwin Brecher

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